# The Philosophy of Open Quantum Systems

Stephan Hartmann (with Mike Cuffaro)

Munich Center for Mathematical Philosophy
LMU Munich

SSLPS Annual Meeting 2018

Lugano, CH
September 2018

# The Closed Systems View

- In standard quantum mechanics, one considers closed systems, i.e. systems which are perfectly isolated from their environment. The resulting dynamics is unitary.
- This is an idealization as physical systems cannot be perfectly isolated from their environment (apart, perhaps, from the whole universe).
- However, the idealization often works remarkably well. For example, the Standard Model of particle physics is very well confirmed.
- This supports...

**The Closed Systems View**

Closed systems are fundamental and any system under consideration is represented as a closed system.

# The Open Systems View

- The closed systems view is deeply entrenched in the methodology of physics: even if open systems (such as lasers) are considered, physicists have developed powerful methods to model the effects of the environment in a closed-system framework.
- The same holds for philosophy of physics: there is almost no work that focuses on philosophical problems of open (quantum) systems.
- To change this unfortunate situation, the goal of this talk is to articulate and defend. . .

**The Open Systems View**

Open systems are fundamental and any system under consideration is represented as an open system.

- On this view, the fact that the system of interest interacts with the external environment is essential to our description of the system.

# What do we mean by “fundamental”?

Here are three necessary conditions for a fundamental theoretical description.

1. Let S be a set of phenomena that is accounted for by two different theories, viz. $T_F$ and $T_P$. $T_F$ is more fundamental than $T_P$ for S if it is less idealized than $T_P$.
2. The fundamental dynamical laws describing a system’s evolution should not depend for their validity on the system’s initial state.
3. The ability to uncover the causal structure of a system through empirical testing is a fundamental demand one might make on any physical theory.

# Plan

1. Open Quantum Systems
2. The Lindblad Equation
3. Intermezzo: Generalized Dicke States and $SU(4)$
4. Is the Lindblad Equation Fundamental?
5. Outlook and Things to Do

# I. Open Quantum Systems

# The Liouville-von Neumann Equation

- Consider the Schrödinger equation for a pure quantum state $|\psi>$:

$$\mathrm{i}|\dot{\psi}>= H\,|\psi>$$

- Introduce the density operator $\rho := |\psi><\psi|$. Then the Schrödinger equation implies the Liouville-von Neumann equation:

$$\dot{\rho} = -\mathrm{i}\,[H,\rho]$$

- Introducing the superoperator $\mathcal{L}_u\rho := -\mathrm{i}\,[H,\rho]$, the Liouville-von Neumann equation can also be written in the form

$$\dot{\rho} = \mathcal{L}_u\rho,$$

which resembles the classical Liouville equation. $\mathcal{L}_u$ governs the unitary dynamics of the system.

# Mixed States

- Consider a system which is with a probability $p_i$ in the pure state $|\psi_i>$ with $p_i < 1$ and $i = 1, \ldots, n$.
- The physics of this system can be accounted for by the density operator

$$\rho = \sum_{i=1}^{n} p_i |\psi_i >< \psi_i| \, .$$

- Note that this state cannot (in general) be expressed in terms of a wave function and there is no Schrödinger equation that describes the dynamics of the corresponding system.
- Once $\rho$ is specified, experimentally observable expectation values can be computed: $< A >= Tr(A\rho)$.

# Coupled Systems

- Consider a quantum system $\mathcal{S}$ which is coupled to a reservoir $\mathcal{R}$:

$$|\Psi>=\frac{1}{\sqrt{2}}\left(|u>|\mathcal{R}_u>+|d>|\mathcal{R}_d>\right)$$

  with the orthogonal states $|u>$ and $|d>$ for $\mathcal{S}$ and the orthogonal states $|\mathcal{R}_u>$ and $|\mathcal{R}_d>$ for $\mathcal{R}$.
- The corresponding density operator $\rho := |\Psi><\Psi|$ is given by

$$\begin{aligned}\rho &= \frac{1}{2}(|u><u|\otimes|\mathcal{R}_u><\mathcal{R}_u|+|u><d|\otimes|\mathcal{R}_u><\mathcal{R}_d| \\ &+ |d><u|\otimes|\mathcal{R}_d><\mathcal{R}_u|+|d><d|\otimes|\mathcal{R}_d><\mathcal{R}_d|)\end{aligned}$$

- We are only interested in $\mathcal{S}$ and are ignorant about the state of $\mathcal{R}$. Hence, we average over the values of $\mathcal{R}$ ("partial trace") and obtain the reduced density operator $P$ for $\mathcal{S}$:

$$\begin{aligned}P &= Tr_{\mathcal{R}}|\Psi><\Psi| \\ &= \frac{1}{2}\left(|u><u|+|d><d|\right)\end{aligned}$$

# Upshot

- Note that the formalism presented so far takes closed systems as basic: $\mathcal{S} + \mathcal{R}$ is closed, and the composite system is accounted for by a wave function.
- One then derives the effective dynamics of the reduced system taking into account that we are ignorant about the state of the reservoir. This requires the specification of a density operator (and not a wave function).

# II. The Lindblad Equation

# Master Equations in Quantum Optics

- Open systems features such as dissipation and pumping play a crucial role in standard quantum optical applications such as lasers. They are also ubiquitous in quantum information theory.
- There are successful semiclassical theories (bases, e.g., on the Fokker-Planck equation), but an account that also works e.g. for small atom numbers requires a full quantum mechanical treatment: A proper quantum theoretical account requires that the environment is modeled quantum mechanically.
- Systems of this kind are (typically) described by a Markovian quantum master equation of the Lindblad form.
- There are two distinct ways to derive the Lindblad equation:
  1. A microscopic and specific derivation which has to be carried out for each type of system under consideration.
  2. A general and abstract derivation which does not relate directly to a particular system.

# Derivation 1

We consider a specific quantum system $\mathcal{S}$ (here: a two-level atom) which is coupled to an environment $\mathcal{R}$ and make the following assumptions:

1. The total Hamiltonian is given by $H = H_S + H_R + H_{SR}$.
2. $\mathcal{S}$ and $\mathcal{R}$ are initially uncorrelated and weakly coupled (i.e. $\mathcal{S}$ does not affect the state of $\mathcal{R}$).
3. The Born-Markov approximation.

We then "trace out" the environment and arrive at the following equation for the reduced density operator $P$ describing the non-unitary dynamics in the quasi-spin formalism:

## The Lindblad Equation for a 2-level Atom

$$\begin{aligned}\dot{P} = \; & -A \quad (\sigma_+\sigma_- \, P + P \, \sigma_+\sigma_- - 2\,\sigma_- \, P \, \sigma_+) \\ & -B \quad (\sigma_-\sigma_+ \, P + P \, \sigma_-\sigma_+ - 2\,\sigma_+ \, P \, \sigma_-)\end{aligned}$$

The basis states are $|1 >= (1,0)^{\mathrm{T}}$ and $|0 >= (0,1)^{\mathrm{T}}$ and the Pauli matrices $\sigma_\pm$ represent the corresponding raising and lowering operators.

# Some Remarks

- This equation is an instance of the general Lindblad equation which has the form

**The Lindblad Equation**

$$\dot{P} = -\mathrm{i}\,[H, P] + \frac{1}{2} \sum_i \left( [L_i\, P, L_i^\dagger] + [L_i, P\, L_i^\dagger] \right) .$$

- Here the superoperators $L_i$ have to be bounded and $\sum_i L_i^\dagger L_i = 1$.
- A similar equation can be microscopically derived for other dynamics (e.g. for the radiation field).
- Note that the Lindblad equation implies that $Tr\dot{P} = 0$, which guarantees probability conservation.

# Derivation 2

This derivation is very general and abstract.

## References:


1. G. Lindblad: On the Generators of Quantum Dynamical Semigroups. *Commun. Math. Phys.* 48 (2): 119 (1976).
2. V. Gorini, A. Kossakowski and E.C.G. Sudarshan: Completely Positive Semigroups of $N$-Level Systems. *J. Math. Phys.* 17 (5): 821 (1976).

# Derivation 2

We consider the general dynamical equation

$$\dot{\rho} = \mathcal{L}\rho.$$

This equation has the formal solution

$$\rho(t) = \Phi_t \, \rho(0),$$

**Three conditions on the map $\Phi_t$ (more on these later):**

1. $\Phi_t$ is linear (uncontroversial).
2. $\Phi_t$ is completely positive.
3. $\Phi_t$ has the semigroup property, i.e. $\Phi_s \, \Phi_t = \Phi_{s+t}$ and $\Phi_0 = 1$.

# Derivation 2

## Theorem

A linear map $\Phi_t$ that satisfies complete positivity and has the semigroup property is of the Lindblad form.

Note:

- The proof of the theorem crucially relies on a result by Kraus according to which a map $\Phi$ is completely positive iff $\Phi(X) = \sum L_i^\dagger X L_i$ with bounded operators $L_i$ that satisfy $\sum L_i^\dagger L_i = 1$.
- The derivation does not tell us how to construct the generators for a concrete physical situation. (But note that the Schrödinger equation also does not tell us what the Hamiltonian is.)
- Important: Probability is conserved also in the case of a non-unitary time evolution.

# Upshot

- The Lindblad equation contains more physics than the Schrödinger equation (which obtains in a limit).
- Which set of superoperators $L_i$ is used depends on the specific physical application under consideration.
- The general form of the quantum master equation for the reduced density operator $P$ is

$$\dot{P} = (\mathcal{L}_u + \mathcal{L}_{n.u.})P.$$

## III. Intermezzo: Generalized Dicke States and $SU(4)$

**Reference:**

S. Hartmann: Generalized Dicke States. *Quantum Information and Computation* 16, No. 15 & 16: 13331348 (2016); arXiv:1201.1732v2 [quant-ph]

# Motivation

- I would like to show that it is (at least sometimes) computationally advantageous if one does not take the non-unitary dynamics of a systems only as a disturbance to a unitary dynamics but starts the investigation from a consideration of it.
- More specifically, I am interested in solving quantum master equations for a finite number of two-level atoms (or qubits).

$$\dot{P} = -\mathrm{i}\,[H, P] + \mathcal{L}_{n.u.}\, P\,.$$

- $P$ is the reduced density operator of the system, $H$ is the Hamiltonian that governs the unitary dynamics, and $\mathcal{L}_{n.u.}$ is a superoperator that describes the non-unitary (resonator mode and atomic) dynamics.
- Such equations are used to address a variety of problems from quantum optics, such as the description of laser systems.
- In quantum information theory, equations of this form are used to study the decoherence of entangled quantum states.

- For a finite number $Z$ of 2-level atoms, the atomic non-unitary dynamics is of the Lindblad form:

$$L_\sigma^{(Z)} P = - \frac{B}{2}(1-s) \sum_{i=1}^{Z} [\sigma_+^{(i)}\sigma_-^{(i)} P + P\, \sigma_+^{(i)}\sigma_-^{(i)} - 2\sigma_-^{(i)} P \sigma_+^{(i)}]$$
$$- \frac{B}{2} s \sum_{i=1}^{Z} [\sigma_-^{(i)}\sigma_+^{(i)} P + P\, \sigma_-^{(i)}\sigma_+^{(i)} - 2\sigma_+^{(i)} P \sigma_-^{(i)}]$$

- Note: the microscopic derivation presupposes that each atom is coupled to a reservoir.
- $\sigma_\pm^{(i)}$ and $\sigma_3^{(i)}$ are the usual Pauli matrices, acting on atom $i$ whose states are represented by $|1>$ or $|0>$.
- $B$ is a decay constant and $s$ is the pumping parameter. It varies from $s = 0$ for pure damping to $s = 1$ for full laser action.

# My Goal. . .

- . . . is to find atomic basis states that can be used to solve quantum master equations that involve such non-unitary terms.
- To achieve this goal, I take an algebraic approach.
- The basic idea is to take the non-unitary part seriously and start from it. That is, we do not take the non-unitary part (as usual) as a disturbance on top of the (supposedly dominant) unitary dynamics.

# Superoperators

- We define:

$$\mathcal{Q}_{\pm}\, P := \sum_{i=1}^{Z} \sigma_{\pm}^{(i)}\, P\, \sigma_{\mp}^{(i)} \quad , \quad \mathcal{Q}_3\, P := \frac{1}{4} \sum_{i=1}^{Z} \left( \sigma_3^{(i)}\, P + P\, \sigma_3^{(i)} \right)$$

- These operators satisfy the $SU(2)$ commutation relations:

$$\begin{aligned} [\mathcal{Q}_+, \mathcal{Q}_-] &= 2\, \mathcal{Q}_3 \\ [\mathcal{Q}_3, \mathcal{Q}_\pm] &= \pm \mathcal{Q}_\pm \end{aligned}$$

- Next, we define the quadratic (Casimir) superoperator

$$\mathcal{Q}^2 = \mathcal{Q}_- \mathcal{Q}_+ + \mathcal{Q}_3^2 + \mathcal{Q}_3$$

and obtain

$$[\mathcal{Q}^2, \mathcal{Q}_3] = 0\,.$$

# More Superoperators

- We define another set of superoperators:

$$\Sigma_{\pm}\, P := \sum_{i=1}^{Z} \sigma_{\pm}^{(i)}\, P\, \sigma_{\pm}^{(i)} \quad , \quad \Sigma_3\, P := \frac{1}{4} \sum_{i=1}^{Z} \left( \sigma_3^{(i)}\, P - P\, \sigma_3^{(i)} \right)$$

- They also satisfy the $SU(2)$ commutation relations:

$$\begin{aligned} [\Sigma_+, \Sigma_-] &= 2\,\Sigma_3 \\ [\Sigma_3, \Sigma_\pm] &= \pm \Sigma_\pm \end{aligned}$$

- Note also that

$$[\mathcal{Q}_i, \Sigma_j] = 0 \quad \forall i, j \in \{\pm, 3\},$$

i.e. the $su(2)$-subalgebras for $\mathcal{Q}$ and $\Sigma$ are "orthogonal".

# And More Superoperators

$$\mathcal{M}_\pm\, P := \sum_{i=1}^{Z} \sigma_\pm^{(i)}\, P\, \frac{1+\sigma_3^{(i)}}{2} \quad , \quad \mathcal{M}_3\, P := \frac{1}{2} \sum_{i=1}^{Z} \sigma_3^{(i)}\, P\, \frac{1+\sigma_3^{(i)}}{2}$$

$$\mathcal{N}_\pm\, P := \sum_{i=1}^{Z} \sigma_\pm^{(i)}\, P\, \frac{1-\sigma_3^{(i)}}{2} \quad , \quad \mathcal{N}_3\, P := \frac{1}{2} \sum_{i=1}^{Z} \sigma_3^{(i)}\, P\, \frac{1-\sigma_3^{(i)}}{2}$$

$$\mathcal{U}_\pm\, P := \sum_{i=1}^{Z} \frac{1+\sigma_3^{(i)}}{2}\, P\, \sigma_\mp^{(i)} \quad , \quad \mathcal{U}_3\, P := \frac{1}{2} \sum_{i=1}^{Z} \frac{1+\sigma_3^{(i)}}{2}\, P\, \sigma_3^{(i)}$$

$$\mathcal{V}_\pm\, P := \sum_{i=1}^{Z} \frac{1-\sigma_3^{(i)}}{2}\, P\, \sigma_\mp^{(i)} \quad , \quad \mathcal{V}_3\, P := \frac{1}{2} \sum_{i=1}^{Z} \frac{1-\sigma_3^{(i)}}{2}\, P\, \sigma_3^{(i)}$$

# The Generators of $SU(4)$

- To simplify notation, let $\mathbf{O} := \{\mathcal{Q}, \Sigma, \mathcal{M}, \mathcal{N}, \mathcal{U}, \mathcal{V}\}$.
- We then note that for all $X \in \mathbf{O}$:

$$\begin{aligned} [X_+, X_-] &= 2\, X_3 \\ [X_3, X_\pm] &= \pm X_\pm \end{aligned}$$

- Note also that

$$[\mathcal{M}_i, \mathcal{N}_j] = [\mathcal{U}_i, \mathcal{V}_j] = 0 \quad \forall i, j \in \{\pm, 3\}.$$

- In total there are 18 superoperators. It turns out that only 15 of them are linearly independent.
- They satisfy the commutation relations of $SU(4)$.

# Basis States

- The fundamental representation of the group $SU(4)$, adapted to the present case, is explicitly given by

$$u := |1 >< 1| \quad , \quad d := |0 >< 0|$$
$$s := |1 >< 0| \quad , \quad c := |0 >< 1|.$$

- Note that $Tr(u) = Tr(d) = 1$ and $Tr(s) = Tr(c) = 0$. (The analogy to the $SU(4)$ quark model is obvious.)
- Let us now construct the higher-order representations. For the configuration $u^\alpha \, d^\beta \, s^\gamma \, c^\delta$ (with $\alpha + \beta + \gamma + \delta = Z$), these are given by

$$P_Y = \mathcal{S}^Y \, (u^\alpha \, d^\beta \, s^\gamma \, c^\delta).$$

- The symmetrizer $\mathcal{S}^Y$ makes sure that the state has symmetry type $Y$. $\mathcal{S}^Y$ is the product of 2-particle symmetrizers and anti-symmetrizers.

# Fully Symmetrical States

- The most important states are the fully symmetrical states. They can be characterized by three quantum numbers. We choose the eigenvalues of $\mathcal{Q}^2$ (i.e. $q(q+1)$), $\mathcal{Q}_3$ (i.e. $q_3$) and $\Sigma_3$ (i.e. $\sigma_3$).
- The states are then represented by $P^{(Z)}_{q,q_3,\sigma_3}$. Here are some examples:

$$\begin{aligned} P^{(3)}_{3/2,3/2,0} &= \mathcal{S}\left(u^3\right) = u^3 = |111><111| \\ P^{(3)}_{1,1,1/2} &= \mathcal{S}\left(u^2 s\right) = \frac{1}{3}\left(u^2 s + usu + su^2\right) \\ &= \frac{1}{3}\,|111>\left(<110| + <101| + <011|\right). \end{aligned}$$

- Applying $\mathcal{Q}_-$ to $P^{(3)}_{3/2,3/2,0}$, we obtain $|011><011| + |101><101| + |110><110| = 3\,P^{(3)}_{3/2,1/2,0}$.

# Dimension of the Basis

For the fully symmetrical subspace, we obtain:

$$D_{GDS}(Z) = \frac{1}{6}(Z+1)(Z+2)(Z+3)$$

- This has to be compared to $4^Z$ if one proceeds by "brute force".
- Note that the proposed states generalize the well-known Dicke states in a natural way. This is why we have chosen the term "Generalized Dicke States" (GDS) for them.

# Solving Quantum Master Equations

- The quantum master equation

$$\begin{aligned}\dot{P} &= -\frac{B}{2}(1-s)\sum_{i=1}^{Z}[\sigma_+^{(i)}\sigma_-^{(i)}P + P\,\sigma_+^{(i)}\sigma_-^{(i)} - 2\sigma_-^{(i)}P\,\sigma_+^{(i)}] \\ &- \frac{B}{2}s\sum_{i=1}^{Z}[\sigma_-^{(i)}\sigma_+^{(i)}P + P\,\sigma_-^{(i)}\sigma_+^{(i)} - 2\sigma_+^{(i)}P\,\sigma_-^{(i)}]\end{aligned}$$

can be compactly expressed as follows (with $\tau := B\,t$):

$$\frac{dP}{d\tau} = [-Z/2 + (1-s)\ \mathcal{Q}_- - (1-2s)\ \mathcal{Q}_3 + s\ \mathcal{Q}_+]\ P$$

- This equation has the formal solution:

$$P(\tau) = e^{-Z/2\,\tau}\, e^{((1-s)\ \mathcal{Q}_- - (1-2s)\ \mathcal{Q}_3 + s\ \mathcal{Q}_+)\,\tau}\, P(0)$$

# Solving Quantum Master Equations

We now use the BCH-formula to factorize the second exponential:

$$P(\tau) = e^{-Z/2\,\tau}\, e^{A_s(\tau)\mathcal{Q}_+}\, e^{B_s(\tau)\mathcal{Q}_3}\, e^{C_s(\tau)\mathcal{Q}_-}\, P(0)$$

with

$$\begin{aligned} A_s(\tau) &= \frac{s\, f(\tau)}{1 - s\, f(\tau)} \\ B_s(\tau) &= -\tau - 2\log(1 - s\, f(\tau)) \\ C_s(\tau) &= \frac{(1-s)\, f(\tau)}{1 - s\, f(\tau)} \end{aligned}$$

and

$$f(\tau) := 1 - e^{-\tau}.$$

# Two Methodological Remarks

1. Include the unitary dynamics. If the Master equation also contains a unitary dynamics (represented, e.g., by the Tavis-Cummings Hamiltonian), then the atomic part of the corresponding terms can also be expressed in terms of the $SU(4)$-generators. To account for the field-part, one can use the damping basis of Briegel and Englert which has a similar motivation as the GDS. This leads to very efficient computations for practical applications.
2. Generality of the proposed account. Note that the derivation of the $SU(4)$-symmetry does not depend on the derivation of the Lindblad equation. The proposed methodology works whenever the Liouville operator can be expressed in terms of the $SU(4)$-generators. This is typically the case when the underlying dynamics of the quasi-spin system is Markovian.

## IV. Speculation: Is the Lindblad Equation Fundamental?

# What do we mean by a fundamental theoretical description?

Some necessary conditions:

1. Let S be a set of phenomena that is accounted for by two different theories, viz. $T_F$ and $T_P$. $T_F$ is more fundamental than $T_P$ for S if it is less idealized than $T_P$.
2. The fundamental dynamical laws describing a system's evolution should not depend for their validity on the system's initial state.
3. The ability to uncover the causal structure of a system through empirical testing is a fundamental demand one might make on any physical theory.

# Is the Lindblad Equation Fundamental?

S. Weinberg (2017):[1]

“Obviously, probabilities must all be positive numbers, and add up to 100 percent. . . .”

[1] Steven Weinberg, “The Trouble with Quantum Mechanics,” *The New York Review of Books*, Jan. 19, 2017.

# Is the Lindblad Equation Fundamental?

S. Weinberg (2017):[1]

"Obviously, probabilities must all be positive numbers, and add up to 100 percent. There is another requirement, satisfied in ordinary quantum mechanics, that in entangled states the evolution of probabilities during measurements cannot be used to send instantaneous signals, which would violate the theory of relativity. ..."

[1] Steven Weinberg, "The Trouble with Quantum Mechanics," *The New York Review of Books*, Jan. 19, 2017.

# Is the Lindblad Equation Fundamental?

S. Weinberg (2017):[1]

"Obviously, probabilities must all be positive numbers, and add up to 100 percent. There is another requirement, satisfied in ordinary quantum mechanics, that in entangled states the evolution of probabilities during measurements cannot be used to send instantaneous signals, which would violate the theory of relativity. Special relativity requires that no signal can travel faster than the speed of light. ..."

[1]Steven Weinberg, "The Trouble with Quantum Mechanics," *The New York Review of Books*, Jan. 19, 2017.

# Is the Lindblad Equation Fundamental?

S. Weinberg (2017):[1]

"Obviously, probabilities must all be positive numbers, and add up to 100 percent. There is another requirement, satisfied in ordinary quantum mechanics, that in entangled states the evolution of probabilities during measurements cannot be used to send instantaneous signals, which would violate the theory of relativity. Special relativity requires that no signal can travel faster than the speed of light. When these requirements are put together, it turns out that the most general evolution of probabilities satisfies an equation of a class known as Lindblad equations . . . "

[1] Steven Weinberg, "The Trouble with Quantum Mechanics," *The New York Review of Books*, Jan. 19, 2017.

# Is the Lindblad Equation Fundamental?

"...The class of Lindblad equations contains the Schrödinger equation of ordinary quantum mechanics as a special case, ..."

---

Steven Weinberg, "The Trouble with Quantum Mechanics," *The New York Review of Books*, Jan. 19, 2017.

# Is the Lindblad Equation Fundamental?

"... The class of Lindblad equations contains the Schrödinger equation of ordinary quantum mechanics as a special case, but in general these equations involve a variety of new quantities that represent a departure from quantum mechanics. ..."

Steven Weinberg, "The Trouble with Quantum Mechanics," *The New York Review of Books*, Jan. 19, 2017.

# Is the Lindblad Equation Fundamental?

"... The class of Lindblad equations contains the Schrödinger equation of ordinary quantum mechanics as a special case, but in general these equations involve a variety of new quantities that represent a departure from quantum mechanics. These are quantities whose details of course we now don't know. Though it has been scarcely noticed outside the theoretical community, there already is a line of interesting papers, going back to an influential 1986 article by [GRW], that use the Lindblad equations to generalize quantum mechanics in various ways."

---

Steven Weinberg, "The Trouble with Quantum Mechanics," *The New York Review of Books*, Jan. 19, 2017.

# Is the Lindblad Equation Fundamental?

- On this view, the Lindblad equation (and *not* the Schrödinger equation or the Dirac equation) is fundamental.
- The density operator $\rho$ is the fundamental quantity, and not the wave function $\psi$.
- A stochastic (i.e. indeterministic) dynamics is fundamental. It suggests that there is always a general non-reducible level of noise in the background of all systems (including the universe as a whole if we describe it quantum mechanically).
- This has interesting consequences (to be explored) for the debate about the interpretation of quantum mechanics.
- However, no-signalling is insufficient to yield quantum theory
  - Popescu-Rohrlich boxes
  - What does Weinberg mean?

# Two Not Uncontroversial Assumptions

Recall that the general dynamical equation

$$\dot{\rho} = \mathcal{L}\rho$$

has the formal solution

$$\rho(t) = \Phi_t \, \rho(0).$$

Impose three conditions on the map $\Phi_t$:

1. $\Phi_t$ is linear.
2. $\Phi_t$ is completely positive.
3. $\Phi_t$ has the semigroup property, i.e. $\Phi_s \, \Phi_t = \Phi_{s+t}$ and $\Phi_0 = 1$.

$\Rightarrow$ Forces $\Phi_t$ to be of Lindblad form.

# Two Not Uncontroversial Assumptions

Recall that the general dynamical equation

$$\dot{\rho} = \mathcal{L}\rho$$

has the formal solution

$$\rho(t) = \Phi_t \, \rho(0).$$

Impose three conditions on the map $\Phi_t$:

1. $\Phi_t$ is linear (uncontroversial).
2. $\Phi_t$ is completely positive.
3. $\Phi_t$ has the semigroup property, i.e. $\Phi_s \, \Phi_t = \Phi_{s+t}$ and $\Phi_0 = 1$.

$\Rightarrow$ Forces $\Phi_t$ to be of Lindblad form.

# Complete Positivity

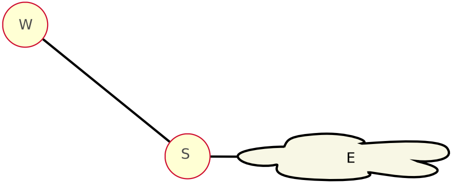
$\Phi_t$: maps density operators to density operators.

Density operators are positive operators.

$\therefore \Phi_t$ must be a positive map.

*Complete* positivity (CP):

- Require $\Phi_t$ to be positive *for all* $\rho_S$
- Even in the presence of a 'witness'



# Complete Positivity

$\Phi_t$: maps density operators to density operators.
Density operators are positive operators.
$\therefore \Phi_t$ must be a positive map.

*Complete* positivity (CP):

- Require $\Phi_t$ to be positive *for all* $\rho_S$
- Even in the presence of a 'witness'

$\rho_{SW}$

- $W$ : 'witness' system. $S, W$ entangled, don't interact.
- $\Phi_t$: evolution of $S$ $I$: evolution (trivial) of $W$

Require that: $\rho_{SW} \xrightarrow{\Phi_t \otimes I} \rho'_{SW}$ be positive for $W$ of any dimension

# Not Completely Positive Maps

Cuffaro & Myrvold (2013):[2]

- $\Phi \otimes I$ should not be formally required to be positive for all $\rho_{SW}$
  - $S, E$ initially entangled: $\rho_S = tr_E(\rho_{SE})$ not pure!
- 'Impossible' states of $S$: $\Phi\rho_S \ngeq 0$,
- 'Possible' states of $S$: $\Phi\rho_S \geq 0$
- 'Not completely positive evolution': misleading terminology:
  - $\Phi$ : *Partial*-CP map w/ NCP extension
  - $\Phi$ : CP with respect to its domain of definition

[2] MEC & and Wayne C. Myrvold (2013), "On the Debate Concerning the Proper Characterisation of Quantum Dynamical Evolution," *Philosophy of Science 80*, 1125–1136.

# Complete Positivity: Upshot

1. One can continue to regard CP dynamics as fundamental despite the pragmatic usefulness of NCP maps in certain situations.
2. The use of NCP maps need not be taken to violate the fundamental requirement that the validity of the dynamical laws describing a system's evolution be independent of the system's initial state.

**Recall:**

Imposing three conditions on the map $\Phi_t$ forces it to take Lindblad form:

1. $\Phi_t$ is linear (uncontroversial).
2. $\Phi_t$ is completely positive (we argued that this is reasonable).
3. $\Phi_t$ has the semigroup property, i.e. $\Phi_s \, \Phi_t = \Phi_{s+t}$ and $\Phi_0 = 1$.
    - This condition amounts to requesting that the (fundamental) dynamics is Markovian.
    - Question: Is this a reasonable requirement?

# Markov

The Markov condition naturally generalizes deterministic causal talk:

- The current state yields the probability distribution for the subsequent state.
- It is unclear how a process can be thought of as causal without it.

It realizes our 'fundamental demand' on any physical theory that it allow for uncovering the causal structure of a system (at least at the surface-level) through empirical testing:

- We need to be able to say how our quantum experiments are related to one another and talk about the causal structure associated with the results of experiments in different labs.
- At any rate it is a feature of how we currently practice (quantum) physics, even if it could be argued to not be strictly necessary.

# Quantum Causal Modelling

This demand can be satisfied within quantum theory:

**References:**


1. Fabio Costa & Sally Shrapnel (2016): "Quantum Causal Modelling," *New Journal of Physics 18*, 063032.
2. John-Mark A. Allen, Jonathan Barrett, Dominic C. Horsman, Ciarán M. Lee, and Robert W. Spekkens (2017): "Quantum Common Causes and Quantum Causal Models," *Physical Review X 7*, 031021.
3. Sally Shrapnel (Advance Access): "Discovering Quantum Causal Models," *British Journal for the Philosophy of Science*.

# Quantum Causal Modelling: Motivation

- The starting point of these investigations is the deterministic interventionist framework of Pearl, Glymour, Spirtes and Scheines, in which the Markov Condition and the Faithfulness Condition (no fine-tuning) are both required to allow for causal discovery.
- To apply the framework to quantum theory (and to deal with the violation of the Bell inequalities), people suggested to give up either Markov or Faithfulness. This may appear to be forced and unnatural and the question is whether there is other ways to provide a quantum causal modeling framework that is inherently indeterministic.
- Question: Can the interventionist framework be generalized to a quantum context?

# Quantum Causal Modelling: The Idea

- Uses an open-systems framework.
- Events are associated with ‘local laboratories’ represented using formalism of quantum operations.
- Causal influences identified with signalling influences.
- Interventions modelled as quantum ‘instruments’, i.e. particular sets of quantum operations.
- Signalling: The probability of an event can be influenced by some intervention.

# Quantum Causal Modelling: Results

1. It is always possible to determine the structure of a Markovian quantum causal model from experimental observations.
2. Fine-tuned models receive vanishingly small probability.
3. Causally ordered non-Markovian models can always be reduced to Markovian models through the introduction of latent laboratories.

# Upshot: Motivation for Markov

- The proposed framework naturally generalizes determinism to the case of an irreducibly probabilistic theory.
- Assuming it allows one to realise the 'fundamental demand' on a theory that it allow us to probe the causal structure of the world (at least at the surface level) in the context of quantum experiments.
- Within the interventionist frameworks of (Costa & Shrapnel 2016, Allen et al. 2017, Shrapnel 2018), it allows one to describe non-Markovian processes as reducible to Markovian ones.

# V. Outlook and Things to Do

# Open Questions

- In more recent work, Weinberg suggested a way to test the hypothesis that the Lindblad equation is fundamental using atomic clocks.
- It might be interesting to derive Bohm-style equations in the open-systems framework. Perhaps there are testable consequences?
- We need more investigations with respect to the status of the Markov assumption as a necessary condition for the concept of cause.
- Not all of the three necessary conditions on fundamentality we expressed have the same status. The motivation for Markov seems to bring in methodological issues related to representation; less so for the first two. This needs further exploration. See, for example: Woodward (2015).†

---

†Woodward (2015), "Methodology, Ontology, and Interventionism," *Synthese 192*, 3577–3599.

# Outlook

1. We have developed the "open systems view" of quantum mechanics.
2. It should be taken more seriously as a fundamental account of quantum dynamics and we offered some speculations as to how one might do so.
3. Taking open systems seriously will have implications for the interpretation of quantum theory which should be explored.
4. We have also shown that the "open systems view" has computational advantages in some contexts.
5. Finally, we identified a new permutation symmetry – $SU(4)$ – that holds in open quantum systems comprising (fermionic) two-level subsystems. Studying the states of mixed symmetry may have observable consequences.

The philosophy of open quantum systems is only at the beginning.

**Thanks for your attention!**