Fundamentals of Nuclear Engineering

Module 7: *Nuclear Chain Reaction Cycle*

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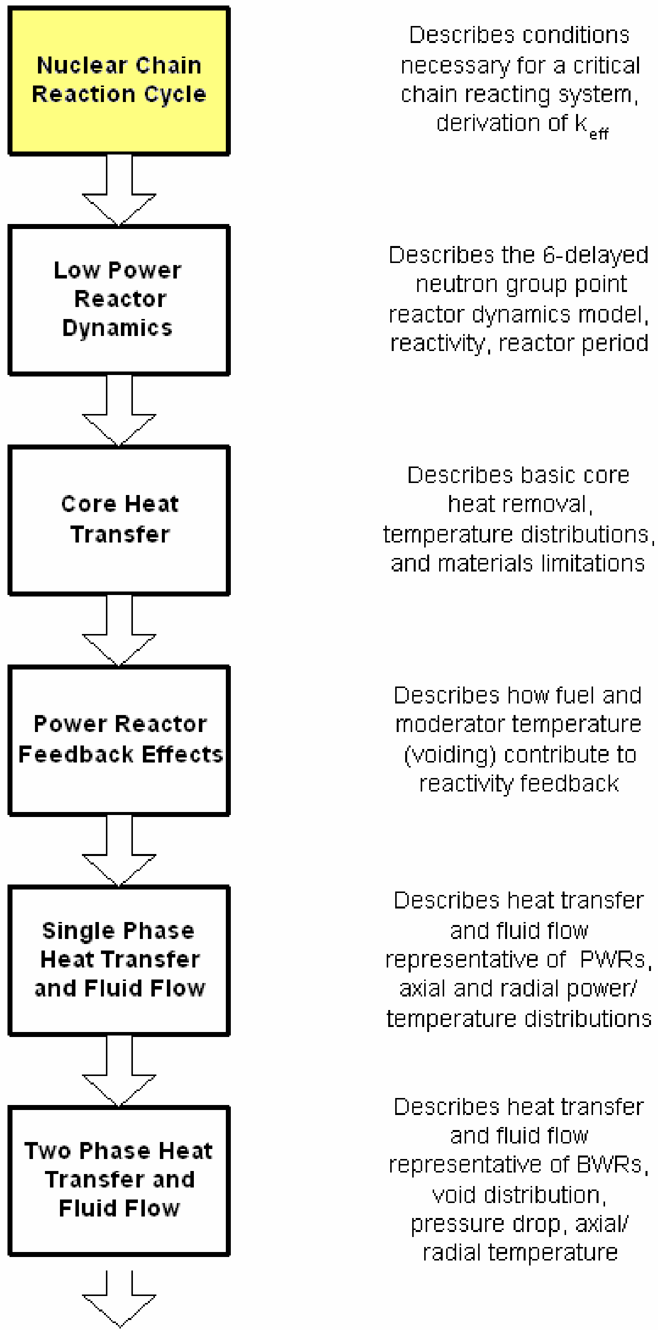
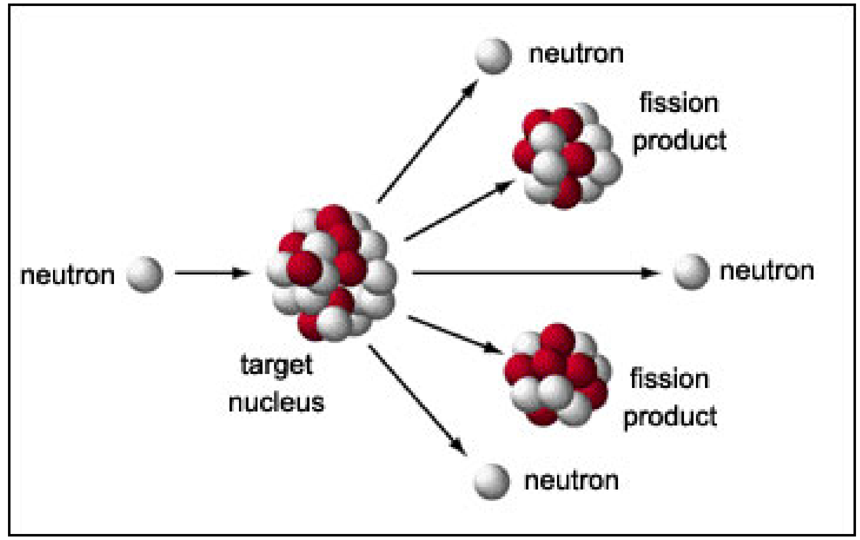

Nuclear Chain Reaction Cycle
Describes conditions necessary for a critical chain reacting system, derivation of k_{eff}
Low Power Reactor Dynamics
Describes the 6-delayed neutron group point reactor dynamics model, reactivity, reactor period
Core Heat Transfer
Describes basic core heat removal, temperature distributions, and materials limitations
Power Reactor Feedback Effects
Describes how fuel and moderator temperature (voiding) contribute to reactivity feedback
Single Phase Heat Transfer and Fluid Flow
Describes heat transfer and fluid flow representative of PWRs, axial and radial power/ temperature distributions
Two Phase Heat Transfer and Fluid Flow
Describes heat transfer and fluid flow representative of BWRs, void distribution, pressure drop, axial/ radial temperature

Objectives:

1. *Define stages of nuclear chain reaction cycle*
2. *Define multiplication factors of reactor systems:*
   - *Subcritical*
   - *Critical*
   - *Supercritical*
3. *Define infinite medium system multiplication factor: k_∞ (four factor formula)*
4. *Define finite medium system multiplication factor: k_{eff} (six factor formula)*
5. *Describe differences in: One-Group, Two-Group, Multi-group core physics calculations*

Chain Reacting Systems

Each Fission produces multiple neutrons:



- Fission yields on average: "*v*" total neutrons
- Fission yield increases *slightly* with neutron energy
- For U^{235}: $v(E) \approx 2.44$
- For U^{233}: $v(E) \approx 2.50$
- For Pu^{239}: $v(E) \approx 2.90$

Multiplication Factor

- Multiplication factor: *"k"* is ratio of current neutron population to previous population
- Nuclear system is:
- *"Subcritical"* if $k < 1.0$ - neutron population decreases in successive generations
- *"Critical"* if $k = 1.0$ - neutron population constant in successive generations
- *"Supercritical"* if $k > 1.0$ - neutron population increases in successive generations

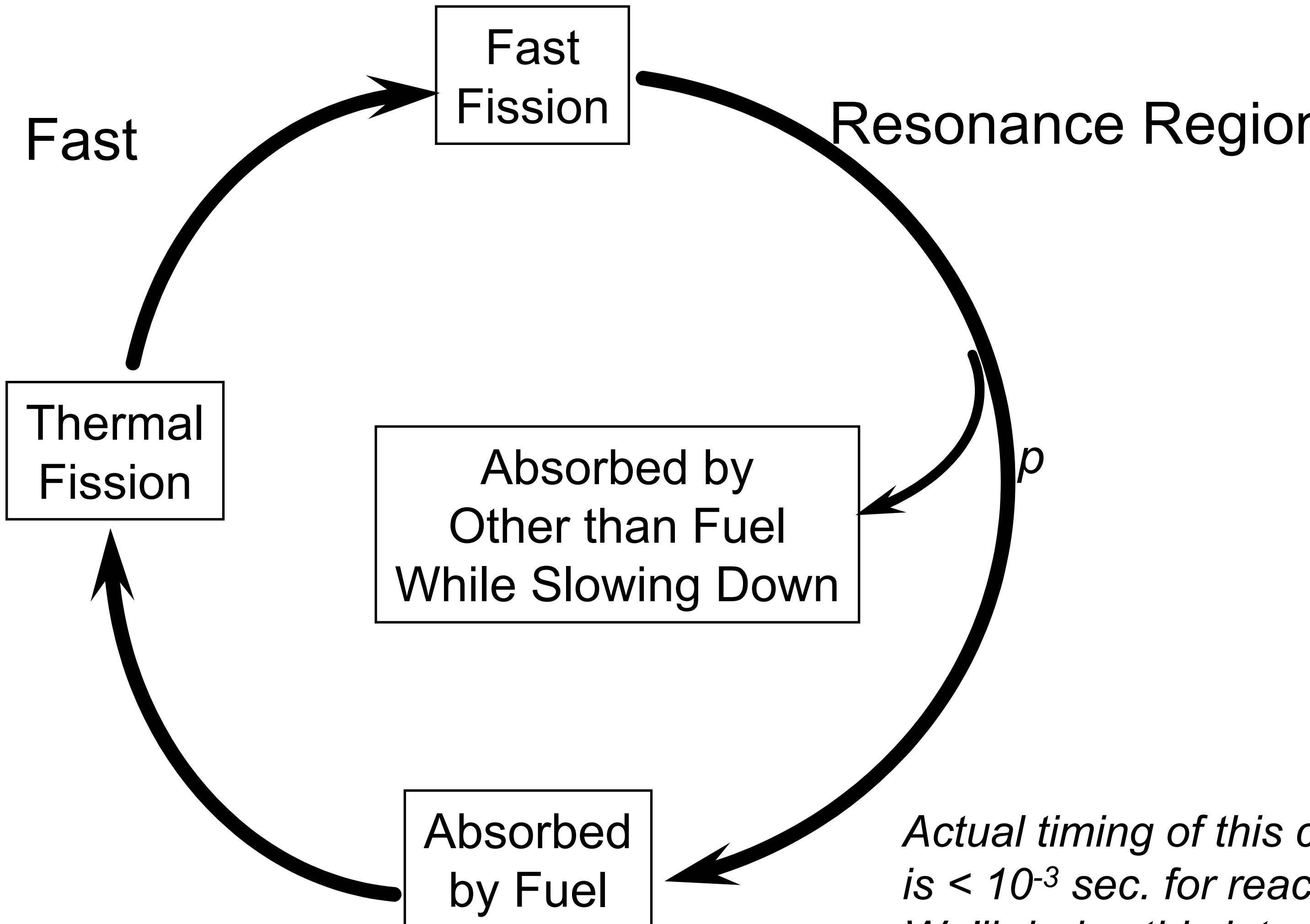
Differences Between: Thermal and Fast Reactors

- Thermal reactors *primarily* rely on *thermal neutrons* to initiate fission
- Thermal reactors include a population of *fast*, *epithermal*, and *thermal neutrons*
- Thermal reactors use some relatively *low A-value* moderator/coolant to slow neutrons down to thermal energy

- Fast reactors rely on *fast neutron fission* processes
- Fast reactors must use *high A-value* coolant (liquid metals)

- Criticality is a measure of *net neutron population*, not energy distribution

Infinite Medium Chain Reaction → No Leakage



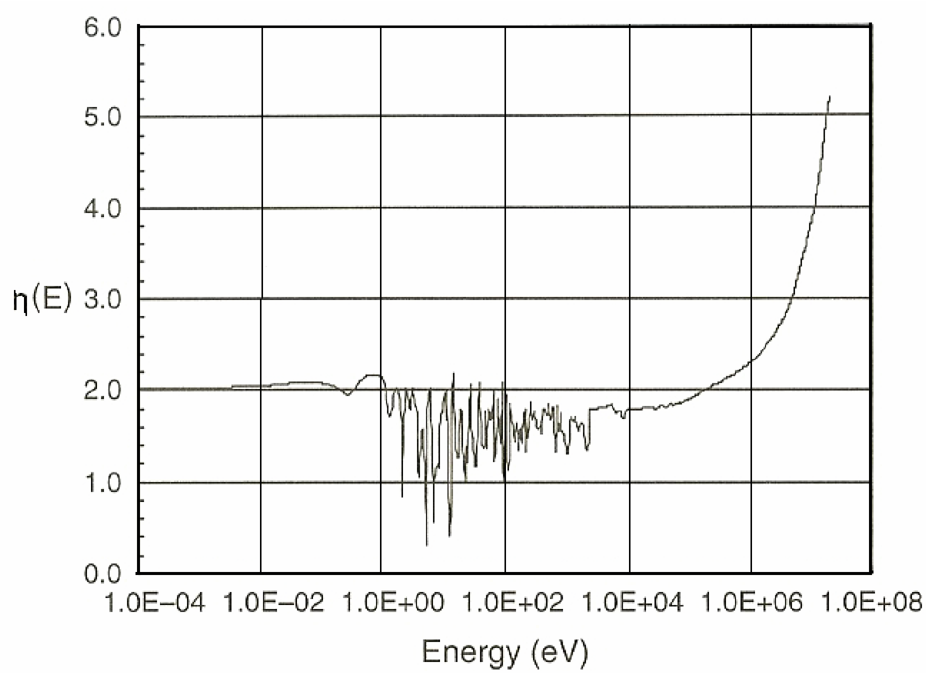
Actual timing of this cycle is < 10^{-3} sec. for reactors. We'll derive this later !

Considering Only Fissile Material

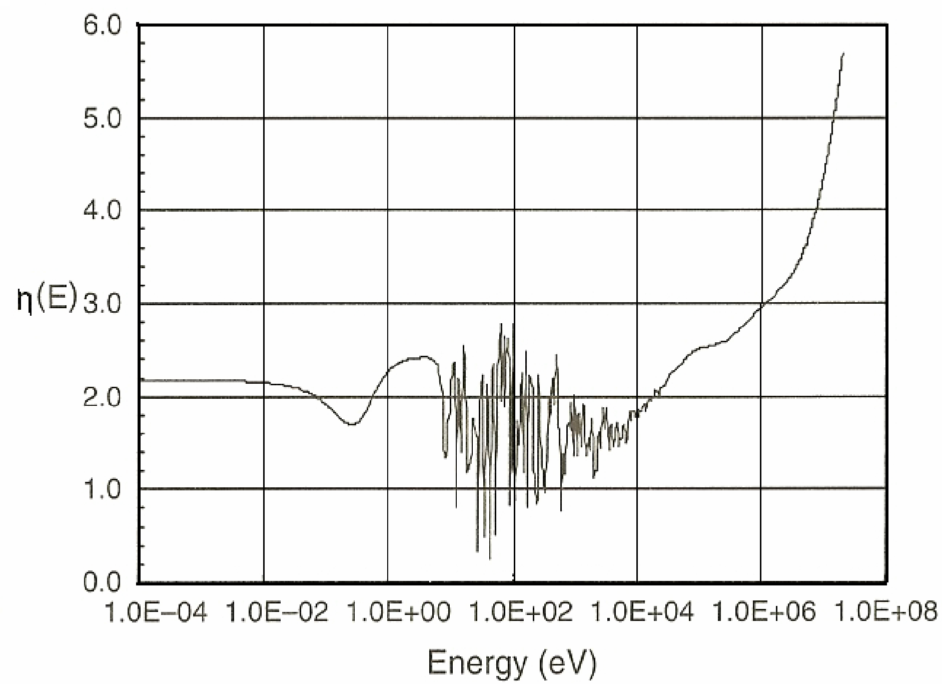
- Ratio of total fission neutrons produced to neutrons absorbed in *infinite medium* is calculated:

$$\eta(E) = \nu(E)\Sigma_f(E) / \Sigma_a(E) = \nu(E)\Sigma_f(E) / (\Sigma_c(E) + \Sigma_f(E))$$

- For one fissile material: $\eta(E) = \nu(E)\sigma_f(E) / (\sigma_c(E) + \sigma_f(E))$
- Examples for pure U^{235} and Pu^{239}



U^{235}



Pu^{239}

Actual Reactor Physics Considerations

- Neutron yield per neutron absorbed "*simply*" defined:

  $\eta(E) = \nu(E)\Sigma_f(E) / \Sigma_a(E) = \nu(E)\Sigma_f(E) / (\Sigma_c(E) + \Sigma_f(E))$

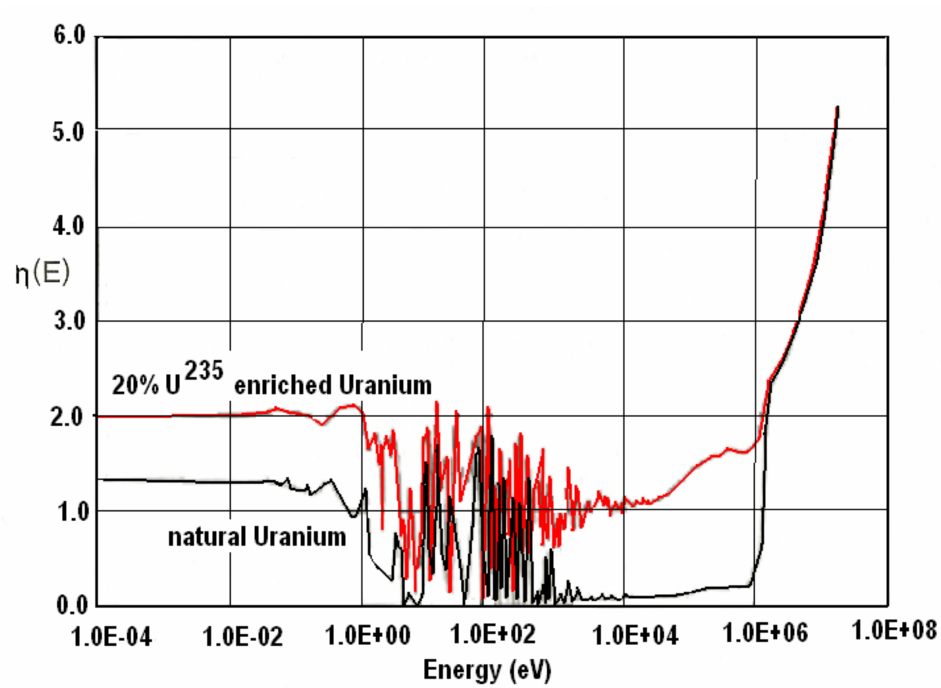
- Actual core physics calculations must consider:
  - All isotopes which capture neutrons: Xe^{135}, Sm^{149}, B^{10}, etc…
  - All isotopes present in fuel that fission: U^{235}, Pu^{239}, Pu^{241}, *etc…*
- Fuel supplier's design would need to consider:
  - Fresh fuel without fission products, Pu^{239}, Pu^{241}
  - Fuel with equilibrium Xe^{135}, Sm^{149}, various buildup of Pu^{239}, Pu^{241}, *etc…*

- For introductory purposes of these lectures we focus on fresh enriched Uranium fuel

Considering Mixture Fissile Material

- Reactor fuel typically mixture of: 2 - 3% U^{235}, U^{238}
- Define enrichment: $e = N_{U235} / (N_{U235} + N_{U238})$

$$\eta(E) = \frac{[e\nu(E)_{U235}\sigma_f(E)_{U235} + (1-e)\nu(E)_{U238}\sigma_f(E)_{U238}]}{[e(\sigma_f(E)_{U235}5 + \sigma_c(E)_{U235}) + (1-e)(\sigma_f(E)_{U238} + \sigma_c(E)_{U238})]}$$



Increasing U^{235} enrichment increases neutron population

from: E. E. Lewis,

"Nuclear Reactor Physics", p. 101

Infinite Medium Multiplication Factor

To generate k_∞ must consider:

- Materials other than fissile fuel
- Cladding
- Coolant/Moderator
- Control Rods
- Structural Materials
- All cause: scattering, thermalizing, capture
- These impact $\varphi(E)$ distribution by:
- Shifting neutron density towards thermal energies
- Depressing neutron density near resonances

Infinite Medium Multiplication Factor

- To generate k_∞ must weight $\nu(E)$ with $\varphi(E)$
- In thermal reactor, cross sections can be *approximated* with thermally averaged values
- This yields:
- k_∞ *approximation* requires corrections for:

Fast fission (adds neutrons)

Resonances (remove neutrons)

Fuel vs. Misc. Absorption

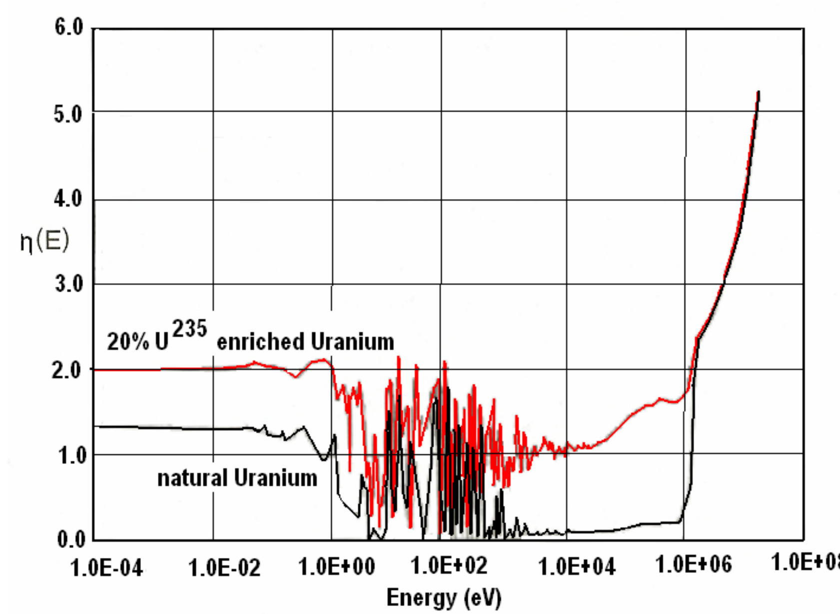
(remove neutrons)

$$k_\infty = \frac{\int_0^\infty \nu(E)\Sigma_f(E)\varphi(E)dE}{\int_0^\infty (\Sigma_c(E) + \Sigma_f(E))\varphi(E)dE}$$

$$k_\infty \approx \frac{\overline{\nu}\overline{\Sigma}_f}{\overline{\Sigma}_c + \overline{\Sigma}_f} = \eta$$

Fast Neutron Fission Correction

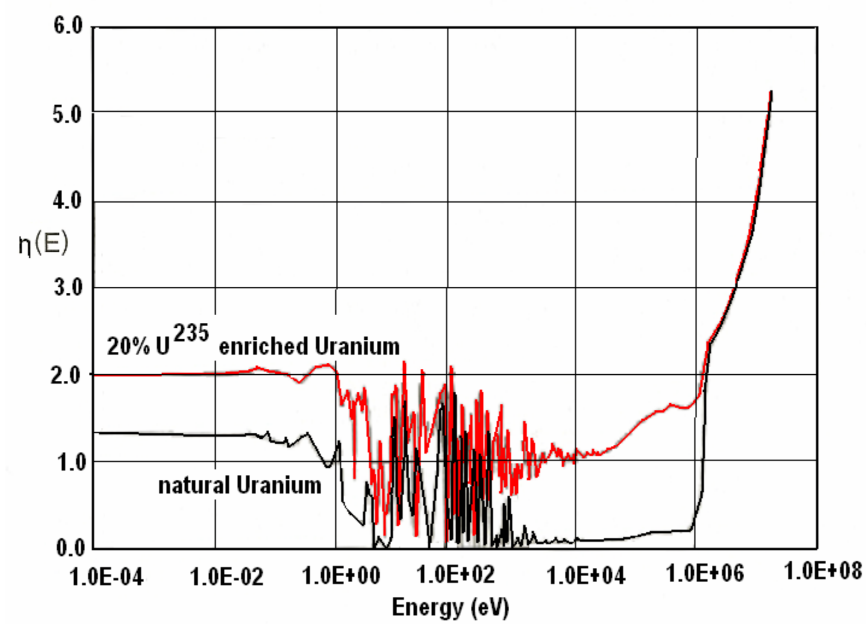
- Given high $\eta(E)$ for fast neutrons, correction factor: ε applied for U^{238}
- ε accounts for additional fissions from fast neutrons
- ε: ratio of total fission neutrons to fission neutrons from thermal neutrons $(E \leq E_t)$ only
- Range: $1.0 \leq \varepsilon \leq 1.227$
- $\varepsilon \approx 1.0$ (if no U^{238} *present)*



$$\varepsilon = \frac{\int_0^\infty \nu(E)\Sigma_f(E)\varphi(E)dE}{\int_0^{E_t} \nu(E)\Sigma_f(E)\varphi(E)dE}$$

Resonance Escape Correction

- Resonance capture in 1eV – 10^4eV range "*depresses*" $\varphi(E)$
- Resonance escape probability: "p" corrects *thermal approximation* "*v*" for neutron losses during thermalization
- Recall *neutron slowing down model*:
- Resonance escape probability models start from this expression

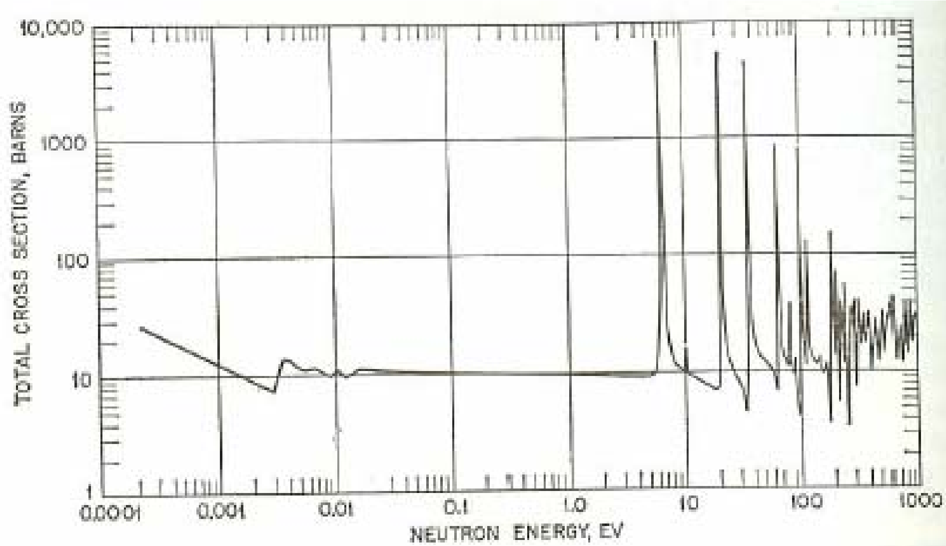


$$\frac{q(E')}{q(E)} = \exp\left[-\int_{E'}^{E} \frac{\Sigma c(E)dE}{\xi(E)(\Sigma c(E) + \Sigma s(E))E} \right]$$

Resonance Escape Correction

- Problem: *Hundreds* of resonances necessitate numerical evaluation or approximation.
- Historical approaches:
- NR - *narrow resonance*
- NRIM - *narrow resonance infinite mass*
- Quasi-experimental p
- Range:

  $p \approx 0.63 - 0.87$ PWR/BWRs (current day designs)



Total cross section of uranium-238 as function of neutron energy

$$p = \exp\left[-\frac{2.73}{\bar{\xi}}\left(\frac{N_A}{N_A\sigma_p + N_m\sigma_m}\right)^{1-0.486}\right]$$

from: J. R. Lamarsh,

"Nuclear Reactor Theory", p. 235

Thermal Utilization Correction

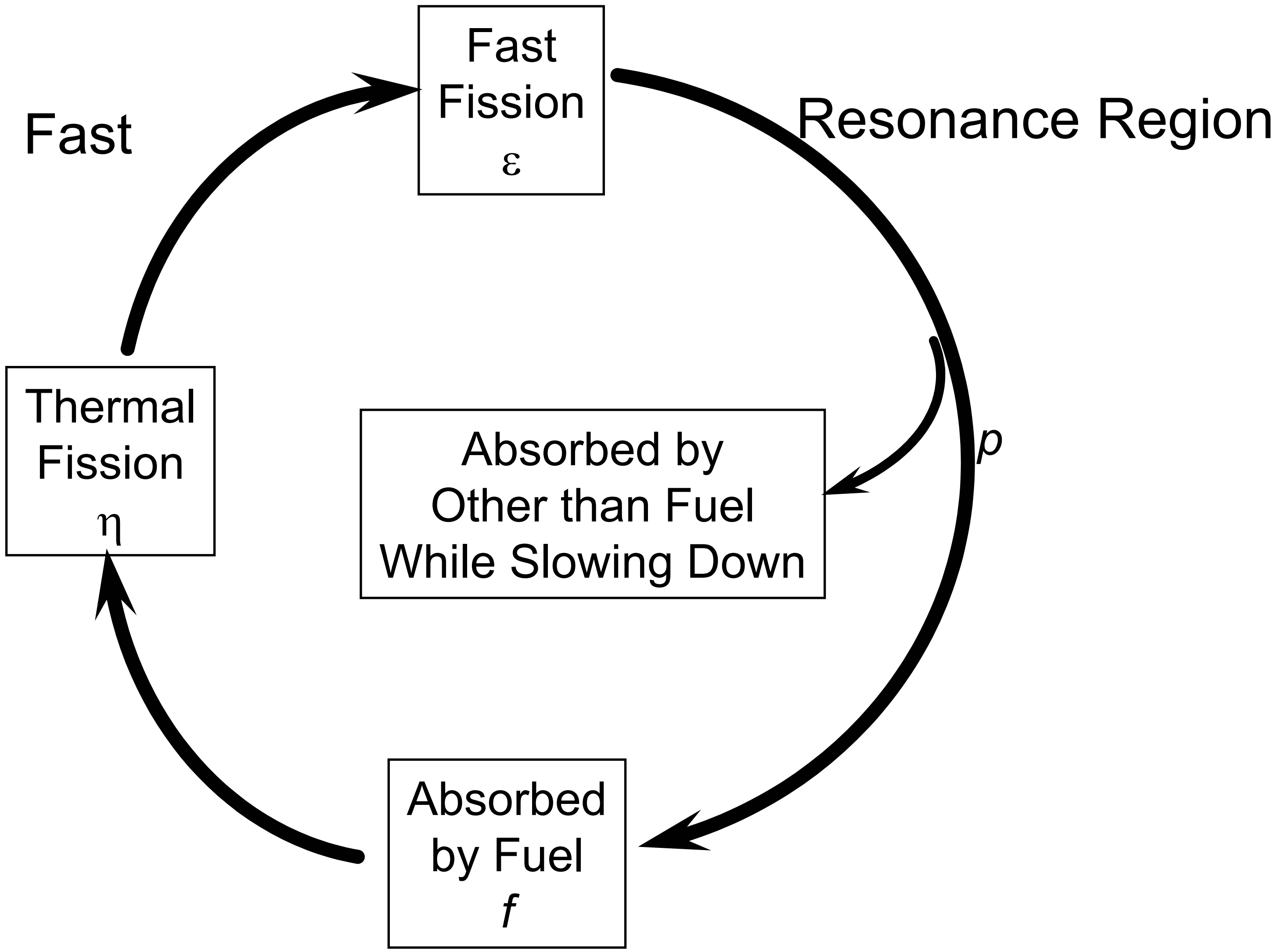
- Thermal neutrons not all absorbed in fuel
- Thermal utilization *"f"* corrects for fraction absorbed in non-fissile materials

$$f = \frac{V_f \int_0^{E_t} (\Sigma_c(E) + \Sigma_f(E))\varphi(E)dE}{V_f \int_0^{E_t} (\Sigma_c(E) + \Sigma_f(E))\varphi(E)dE + V_m \int_0^{E_t} \Sigma_c(E)\varphi(E)dE}$$

$$f = \frac{V_f(\overline{\Sigma}_c + \overline{\Sigma}_f)\overline{\varphi_f}}{V_f(\overline{\Sigma}_c + \overline{\Sigma}_f)\overline{\varphi}_f + V_m\overline{\Sigma}_c\overline{\varphi}_m}$$

- Typical value: $f \approx 0.94$ for PWR/BWR (current day designs)

Infinite Medium Chain Reaction → No Leakage

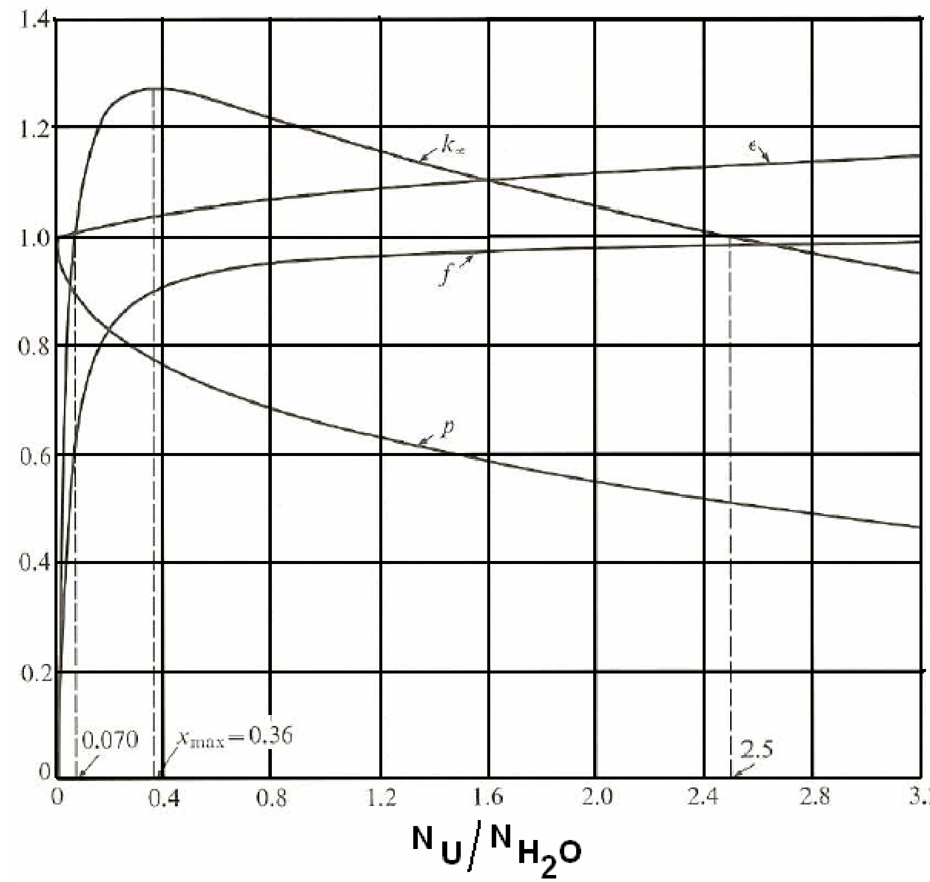


Optimization of Fuel Assembly Design

Effect of Parametrically Varying U-H_2O Ratio

- Assume ~2% Uranium fuel
- Vary Uranium/Water Ratio
- Calculate $\varepsilon, p, f, k_\infty$ as function of: N_U/N_{H2O} ratio
- Fast fission, ε, increases with more U^{238}
- Resonance escape factor, p, decreases with more U
- Thermal utilization, f, levels off after $N_U/N_{H2O} = 1.2$
- η is function of Uranium Σ_c, Σ_f
- Maximum k_∞ is for:

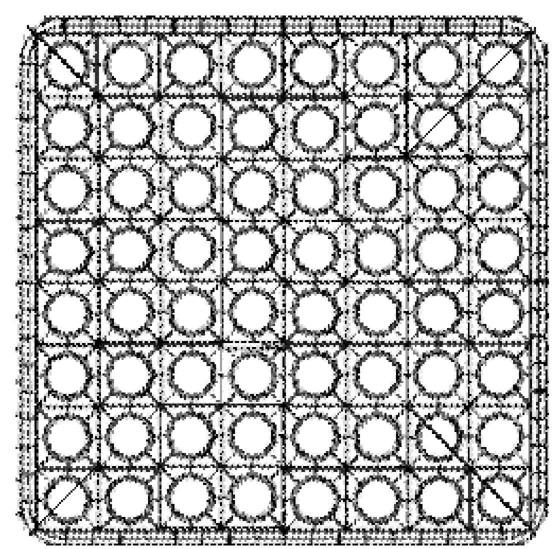
  $N_U/N_{H2O} = 0.36$

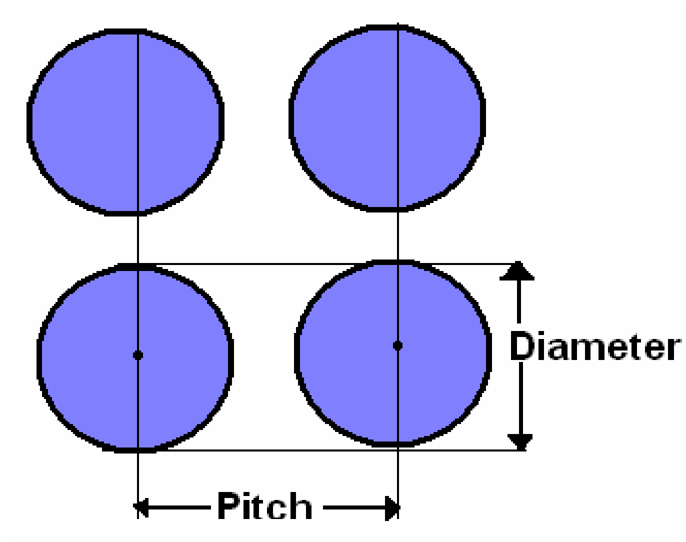


J. Lamarsh, "<u>*Nuclear Reactor Theory*</u>", p. 305

Effect of Core Lattice Geometry on k_∞

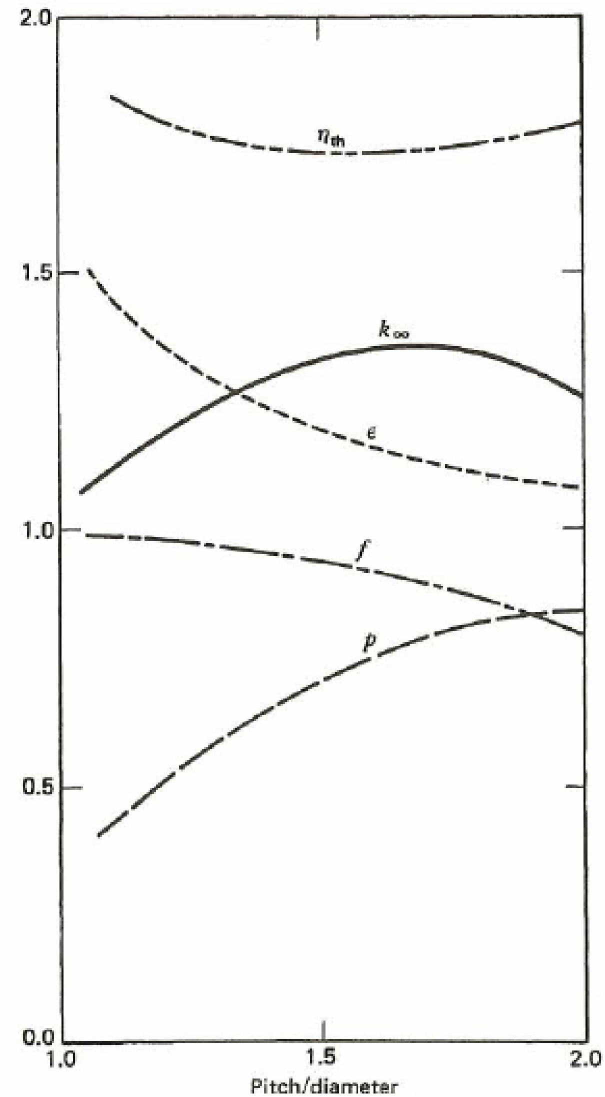
- Reactors are not designed with homogeneous fuel and moderator mixtures
- Typical BWR 8x8 fuel bundle:
- Ratio of water to Uranium is frequently characterized by:
- Pellet Diameter
- Fuel Rod Pitch (center to center distance of fuel pellets)
- Studies have been performed to optimize water to Uranium mixture and geometry





Effect of Core Lattice Geometry on k_∞

- Assume 2-3% Uranium
- Vary fuel pin pitch/diameter ratio
- Calculate $\eta, \varepsilon, p, f, k_\infty$ as function of: pitch/diameter ratio
- Increased pitch increases water:
- Decreases fast fission of U^{238}: ε
- Decreases thermal utilization: f
- Increases resonance escape: p
- k_∞ reaches maximum value at pitch/diameter ≈ 1.65



Effect of fuel lumping on k_∞.

From: J.J.Duderstadt, L.J. Hamilton,

“Nuclear Reactor Analysis, p.405

Homogenous vs. Heterogeneous

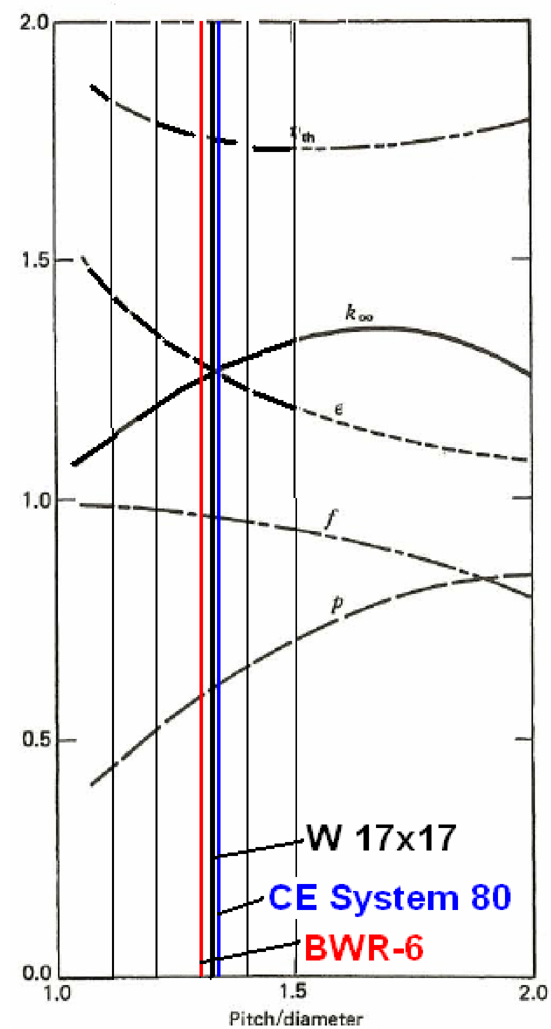
- Homogenous reactor system would be uniform mixture of fuel, moderator, absorbers, and poison
- As: p, f factors tend to completely homogenous mixture:
    - $p \to 1.0$ (due to faster moderation, less resonance capture)
    - But: f decreases (due to parasitic capture in light water)
- Recall:

$$f = \frac{V_f(\overline{\Sigma}_c + \overline{\Sigma}_f)\overline{\varphi_f}}{V_f(\overline{\Sigma}_c + \overline{\Sigma}_f)\overline{\varphi}_f + V_m\overline{\Sigma}_c\overline{\varphi}_m}$$

- Early experiments and calculations showed that separating fuel from moderator allowed minimum critical dimensions to be reduced for light water reactors

Comparisons to Actual Vendor Fuel Designs

Vendor: Type:	GE BWR-6	W RESAR	B&W	CE System 80
Bundle Array:	8x8	17x17	17x17	16x16
U^{235}%	2.2-2.7	2.1-3.1	2.91	1.9-2.9
Pitch:	1.62cm.	1.25cm.	1.27cm.	1.28cm.
Pellet Diameter:	1.25cm.	0.94cm.	0.96cm.	0.97cm.
Pitch : Diameter	1.30	1.32	1.32	1.33



Effect of fuel lumping on k_∞.

Four Factor Formula for: k_∞

- Infinite medium multiplication factor
- Using *Thermal Averaged Approximations*:
- $k_\infty = \eta \varepsilon p f$
- Typical ranges, fresh fuel (no poison/shims):

Parameter	PWR	BWR
η	1.65 -1.89	1.65 - 1.89
ε	1.02 - 1.27	1.02 - 1.28
p	0.63 – 0.87	0.63 – 0.87
f	0.71 - 0.94	0.71 - 0.94
k_∞	1.04 - 1.41	1.04 - 1.40

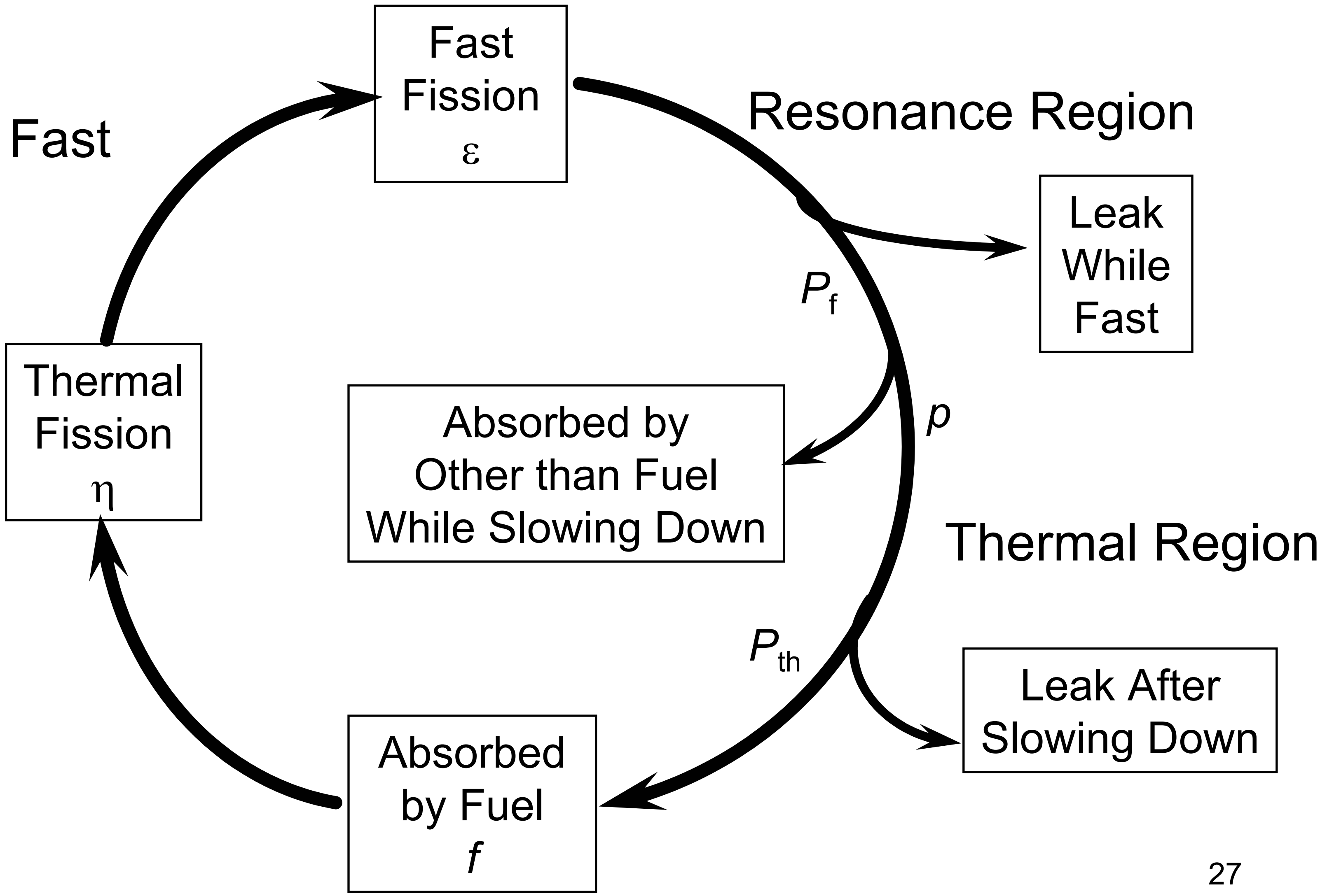
from: E. E. Lewis, "Nuclear Reactor Physics", p. 101,

J.J. Duderstadt, L.J. Hamilton, "Nuclear Reactor Analysis", p. 83.

Reactors Not Infinite-Medium Systems

- In ideal infinite medium: *no surface/volume effects*
- Fast and Thermal leakage out of chain reacting region needs to be considered in finite systems
- Leakage effects result in: "k_{eff}"
- Effective multiplication factor k_{eff} is derived from k_∞ via adjustments for leakage effects
- Thus: $k_{eff} = k_\infty P_f P_{th}$
- Where:
- P_f corrects k_∞ for fast neutron leakage
- P_{th} corrects k_∞ for thermal neutron leakage

Finite Medium Chain Reaction → Leakage



One Group Diffusion Criticality Model

One-Group Diffusion Criticality Model

- Assume that all neutrons in bare (non-reflected) reactor are *thermal* – including fission neutrons
- $P_f \approx 1.0$ – no fast neutron leakage
- $k_{eff} = k_\infty P_{th}$
- P_{th} can be determined from One-Group Neutron Diffusion Model and solving for Eigenvalues that yield an assumed Critical condition

One-Group Diffusion Criticality Model

- Assume steady-state “bare” critical reactor system (no reflected neutrons)

$$0 = S(\vec{r}) - \phi(\vec{r})\Sigma_a(\vec{r}) + D\nabla^2\phi(\vec{r})$$

- Assume source is from thermal neutron fission:

$$S(\vec{r}) = \Sigma_a(\vec{r})\phi(\vec{r})k_\infty$$

- Rearrange by dividing out absorption cross section and flux:

$$0 = k_\infty - 1 + \frac{D\nabla^2\phi(\vec{r})}{\Sigma_a(\vec{r})\phi(\vec{r})} = k_\infty - 1 + L^2\frac{\nabla^2\phi(\vec{r})}{\phi(\vec{r})}$$

- Recognize that Geometrical Buckling: *B* is eigenvalue of:

$$\frac{\nabla^2\phi(\vec{r})}{\phi(\vec{r})} = -B^2$$

- Given *assumption* of critical system, following constraint exists defining relationship for criticality:

$$0 = k_\infty - 1 - L^2B^2$$

$$\frac{k_\infty}{1 + L^2B^2} = 1$$

One-Group Diffusion Criticality Model

- For finite medium, k_{eff} can be defined:

$$k_{eff} = \frac{k_\infty}{1 + L^2 B^2}$$

- The thermal non-leakage probability P_{th} is thus:

$$P_{th} = \frac{1}{1 + L^2 B^2}$$

Example: Yankee Rowe – Fresh Fuel

Based upon Yankee Rowe core with SS Clad, 2.7% enriched U^{235}

$\Sigma aU235 := 0.132$	Average macroscopic neutron absorbtion cross section in U^{235} in cm^{-1}.
$\Sigma aU238 := 0.0192$	Average macroscopic neutron absorbtion cross section in U^{238} in cm^{-1}.
$\Sigma aH2O := 0.0131$	Average macroscopic neutron absorbtion cross section in H_2O in cm^{-1}.
$\Sigma aClad := 0.0180$	Average macroscopic neutron absorbtion cross section in Clad in cm^{-1}.
$\Sigma fU235 := 0.1113$	Average macroscopic fission cross section in U^{235} in cm^{-1}.
$\nu := 2.43$	Average number of neutrons generated per U^{235} fission
$\Phi mu := 1.12$	Ratio of: $\frac{\phi m}{\phi u}$
$\Phi cu := 1.06$	Ratio of: $\frac{\phi c}{\phi u}$
$Ho := 700$	Height of Cylindrical Reactor in cm.
$Ro := 150$	Radius of Cylindrical Reactor in cm.

from: S. Glasstone & A. Sesonske, “Nuclear Reactor Engineering” (1967), p. 203

Example: Yankee Rowe – Fresh Fuel

$$\eta := \frac{\nu \cdot \Sigma fU235}{\Sigma aU235 + \Sigma aU238} \qquad \eta = 1.789$$

Neutrons produced per fission / Neutrons absorbed in Uranium

$\varepsilon := 1.044$ Fast fission factor

$p := 0.931$ Resonance escape probability

$$f := \frac{\Sigma aU235 + \Sigma aU238}{\Sigma aU235 + \Sigma aU238 + \Sigma aH2O \cdot \Phi mu + \Sigma aClad \cdot \Phi cu} \qquad f = 0.818$$

Thermal utilization factor

$\eta \cdot f = 1.462$

$L := 2.37$ Square Root of Diffusion area: $\sqrt{\frac{D}{\Sigma a}}$

$$B := \sqrt{\left(\frac{2.405}{Ro}\right)^2 + \left(\frac{\pi}{Ho}\right)^2} \qquad B = 0.017 \qquad B^2 = 2.772 \times 10^{-4}$$

Geometrical Buckling Factor

$$Pth := \frac{1}{1 + L^2 \cdot B^2} \qquad Pth = 0.998$$

Calculation of Infinite Medium Multiplication Factor

$$k_\infty := \eta \cdot \varepsilon \cdot p \cdot f \qquad k_\infty = 1.421$$

Infinite Medium Multiplication Factor

Calculation of 1-Group k_{eff} Multiplication Factor

$$keff1G := \frac{k_\infty}{1 + L^2 \cdot B^2} \qquad keff1G = 1.419$$

1-Group k_{eff} Multiplication Factor

from: S. Glasstone & A. Sesonske, “Nuclear Reactor Engineering” (1967), p. 204-208

Two-Group Diffusion Criticality Model

Two-Group Diffusion Criticality Model

- Assume that all neutrons in bare (non-reflected) reactor are either: *thermal* or *fast*
- P_f calculated instead of being ignored
- $k_{eff} = k_\infty P_f P_{th}$
- P_f, P_{th} can be determined from Two-Group Neutron Diffusion Model and solving for Eigenvalues that yield an assumed Critical condition.
- $k_\infty = \eta\varepsilon pf$ needs to be split up into portions representing *thermal* (ηf) and *fast* (εp) neutron contributions.

Two-Group Diffusion Criticality Model

- Assume steady-state "bare" critical reactor system (no reflected neutrons) is represented by system of equations:

$$0 = S_f - \phi_f \Sigma_{a-f} + D_f \nabla^2 \phi_f$$

$$0 = S_{th} - \phi_{th} \Sigma_{a-th} + D_{th} \nabla^2 \phi_{th}$$

- Assume fast neutron source is from thermal neutron fission:

$$S_f = \Sigma_{a-th} \phi_{th} \eta f = \frac{D_{th}}{L_{th}^2} \phi_{th} \eta f$$

- Assume thermal neutron source is thermalized fission neutrons enhanced by fast fission effect and which escape resonance capture:

$$S_{th} = \Sigma_{a-f} \phi_f \varepsilon p = \frac{D_f}{L_f^2} \phi_f \varepsilon p$$

Two-Group Diffusion Criticality Model

- Making substitutions and rearranging yields:

$$0 = \left(\frac{D_{th}}{D_f}\right)\frac{1}{L_{th}^{\ 2}}\phi_{th}\eta f - \frac{1}{L_f^{\ 2}}\phi_f + \nabla^2\phi_f$$

$$0 = \left(\frac{D_f}{D_{th}}\right)\frac{1}{L_f^{\ 2}}\phi_f \varepsilon p - \frac{1}{L_{th}^{\ 2}}\phi_{th} + \nabla^2\phi_{th}$$

- Making substitution for geometric Buckling:

$$0 = \left(\frac{D_{th}}{D_f}\right)\frac{1}{L_{th}^{\ 2}}\phi_{th}\eta f - \frac{1}{L_f^{\ 2}}\phi_f - B_f^{\ 2}\phi_f$$

$$0 = \left(\frac{D_f}{D_{th}}\right)\frac{1}{L_f^{\ 2}}\phi_f \varepsilon p - \frac{1}{L_{th}^{\ 2}}\phi_{th} - B_{th}^{\ 2}\phi_{th}$$

Two-Group Diffusion Criticality Model

- This is system of linear equations:

$$\begin{bmatrix} -B_f^{\,2} - \dfrac{1}{L_f^{\,2}} & \left(\dfrac{D_{th}}{D_f}\right)\dfrac{\eta f}{L_{th}^{\,2}} \\ \left(\dfrac{D_f}{D_{th}}\right)\dfrac{\varepsilon p}{L_f^{\,2}} & -B_{th}^{\,2} - \dfrac{1}{L_{th}^{\,2}} \end{bmatrix} \times \begin{bmatrix} \phi_f \\ \phi_{th} \end{bmatrix} = 0$$

- Solving Determinant yields:

$$\begin{vmatrix} -B_f^{\,2} - \dfrac{1}{L_f^{\,2}} & \left(\dfrac{D_{th}}{D_f}\right)\dfrac{\eta f}{L_{th}^{\,2}} \\ \left(\dfrac{D_f}{D_{th}}\right)\dfrac{\varepsilon p}{L_f^{\,2}} & -B_{th}^{\,2} - \dfrac{1}{L_{th}^{\,2}} \end{vmatrix} = (B_f^{\,2} + \frac{1}{L_f^{\,2}})(B_{th}^{\,2} + \frac{1}{L_{th}^{\,2}}) - \eta\varepsilon p f = 0$$

- Which simplifies to:

$$\frac{k_\infty}{(1 + L_f^{\,2} B_f^{\,2})(1 + L_{th}^{\,2} B_{th}^{\,2})} = 1$$

Two-Group Diffusion Criticality Model

- For finite medium, k_{eff} can be defined:

$$k_{eff} = \frac{k_\infty}{(1 + L_f{}^2 B_f{}^2)(1 + L_{th}{}^2 B_{th}{}^2)}$$

- The fast non-leakage probability P_f is thus:

$$P_f = \frac{1}{1 + L_f{}^2 B_f{}^2}$$

- The thermal non-leakage probability P_{th} is thus:

$$P_{th} = \frac{1}{1 + L_{th}{}^2 B_{th}{}^2}$$

Two-Group Criticality Model – Example

- *Thermal multiplication factor:* $\eta = 1.65$
- *Fast fission factor:* $\varepsilon = 1.02$
- *Resonance escape factor:* $p = 0.87$
- *Thermal utilization factor:* $f = 0.71$

- $k_\infty = \eta\varepsilon pf = (1.65)(1.02)(0.87)(0.71) = 1.0396$

- *Fast non-leakage factor:* $P_f = 0.98$
- *Thermal non-leakage factor:* $P_{th} = 0.99$

- $k_{eff} = k_\infty P_f P_{th} = (1.0396)(0.97)(0.99) = 1.008$

Geometrical Buckling

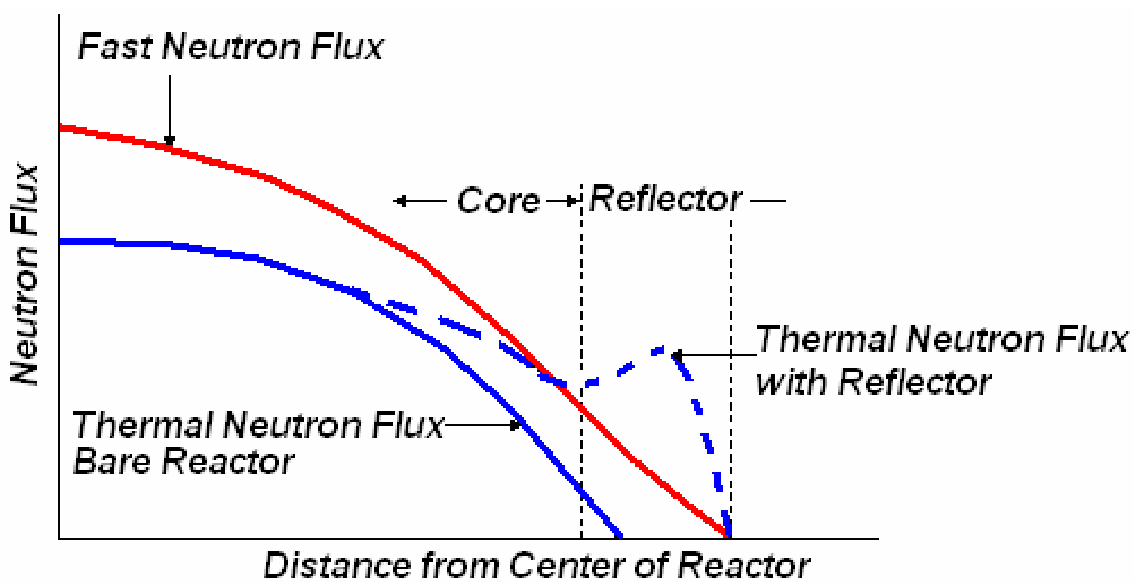
- Geometrical Buckling factor: B^2 is an eigenvalue of Helmholtz type partial differential equation
- Geometrical Buckling factor captures surface to volume effects of different geometries
- Following Buckling factors are for bare, un-reflected core designs:

Geometry:	Dimensions:	Buckling:	Flux Shape:
Rectangular Block	$a \times b \times c$	$B^2 = \left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2 + \left(\frac{\pi}{c}\right)^2$	$\phi(x,y,z) = A_0 \cos\left(\frac{\pi x}{a}\right)\cos\left(\frac{\pi y}{b}\right)\cos\left(\frac{\pi z}{c}\right)$
Sphere	$Radius : R$	$B^2 = \left(\frac{\pi}{R}\right)^2$	$\phi(r) = \frac{A_0}{r}\sin\left(\frac{\pi r}{R}\right)$
Cylinder	$Radius : R$ $Height : H$	$B^2 = \left(\frac{2.405}{R}\right)^2 + \left(\frac{\pi}{H}\right)^2$	$\phi(r,z) = A_0 J_0\left(\frac{2.405 r}{R}\right)\cos\left(\frac{\pi z}{H}\right)$

Taken from: J. Lamarsh, “Nuclear Reactor Analysis, p.298

Effect of Neutron Reflector on Criticality

- Previous discussion of Two-Group Diffusion model noted impact of water region outside of active core.
- Neutron reflection alters the Buckling coefficients derived for *bare, un-reflected* core geometry



Summary Thoughts on Criticality Evaluation:

- Subcriticality, Criticality, Supercriticality conditions are based upon overall "k_{eff}"
- Fuel enrichment, bundle geometry, Uranium to Water ratio directly influences: k_∞
- Fresh fuel bundles (neglecting impacts of poisons or control rods) generally have range of $k_\infty \sim 1.2$ or higher to provide fuel for multiyear power operation
- Overall geometry of core (height, radius), reflector region impact fast and thermal non-leakage probabilities and thus: k_{eff}
- Classical methods described, reflect correct trends, BUT:
- Actual core design process is computer code intensive