

Acquisition, Doppler Tracking, and Positioning with Starlink LEO Satellites: First Results

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Abstract—This letter shows the first acquisition, Doppler tracking, and positioning results with Starlink’s low Earth orbit (LEO) satellite signals. A generalized likelihood ratio (GLR)-based test is proposed to acquire Starlink’s downlink signals. A Kalman filter (KF)-based algorithm for tracking the Doppler frequency from the unknown Starlink signals is developed. Experimental results show Doppler tracking of six Starlink satellites, achieving a horizontal positioning error of 10 m.

Index Terms—signals of opportunity, matched subspace detector, Doppler positioning, low Earth orbit, Starlink.

I. INTRODUCTION

Theoretical and experimental studies have demonstrated the potential of low Earth orbit (LEO) broadband communication satellites as promising reliable sources for navigation [1]–[4]. Companies like Amazon, Telesat, and SpaceX are deploying so-called *megaconstellations* to provide global broadband internet [5]. In particular, launching thousands of space vehicles (SVs) into LEO by SpaceX can be considered as a turning-point in the future of LEO-based navigation technologies. Although they suffer from higher Doppler effect, signals received from LEO SVs can be about 30 dB stronger than signals received from medium Earth orbit (MEO) SVs, where global navigation satellite systems (GNSS) SVs reside [4].

Research has shown that one could exploit LEO SV broadband communication signals opportunistically for navigation purposes [3]. Three of the main challenges of navigation with Starlink SV signals are (i) limited information about the signal structure, (ii) very-high dynamics of Starlink LEO SVs, and (iii) poorly known ephemerides. Assuming that Starlink LEO SV downlink signals contains a periodic reference signal (RS), this paper tackles the first challenge by formulating a *matched subspace detection* problem to (i) detect the unknown RS of Starlink SVs and (ii) estimate the unknown period and Doppler frequency. The second challenge is addressed by adopting a second-order model to capture the dynamics of the Doppler frequency, and designing a Kalman filter (KF)-based algorithm which is capable of tracking the unknown parameters of the Doppler model. A blind approach was presented in [6], [7] to

exploit partially known signals for navigation purposes. However, these approaches were designed for M-ary phase-shift keying (MPSK) signaling and are incapable of deciphering sophisticated signals, such as Starlink’s orthogonal frequency-division multiple access (OFDMA) signals.

This letter makes the following contributions. First, a model for the Starlink LEO SV’s downlink signals is presented. Second, an algorithm is proposed to (i) acquire the Starlink LEO SV signals and (ii) track the Doppler frequency of each detected SV. Third, next to [8], the first experimental positioning results with Starlink downlink signals are presented in this paper. In [8], an adaptive Kalman filter is used to track the carrier phase of Starlink LEO SVs. However, the method presented in [8] relies on tracking the phase of a single carrier. When a more complicated signal structure is used in the downlink signal, e.g., OFDMA, a more sophisticated method should be developed to exploit the entire signal bandwidth for navigation purposes. Indeed, the method in [8] is not capable of exploiting the entire signal bandwidth, and it only relies on tracking a single frequency component. In this paper, by considering a general model for the Starlink downlink signals, the unknown parameters of the signal are estimated for the first time for Starlink LEO SVs, and are subsequently used to detect the Starlink LEO SVs and track their corresponding Doppler frequencies. The proposed method enables one to estimate the synchronization signals of the Starlink LEO SVs.

II. RECEIVED SIGNAL MODEL

A. Starlink Downlink Signals

Except for the carrier frequencies and the bandwidths, more detailed signal specifications of Starlink downlink signals are unavailable to the public. SpaceX uses the Ku-band spectrum for the satellite-to-user links (both uplink and downlink) and the satellite-to-ground contacts are carried out in Ka-band [9]. Software-defined radios (SDRs) allow one to sample bands of the radio frequency spectrum. However, Ku/Ka-bands are beyond the carrier frequency of most commercial SDRs. Hence, in the experiments carried out in this letter, a 10 GHz mixer is employed between the antenna and the SDR to downconvert Starlink LEO SV signals from the Ku-band, namely 11.325 GHz to 1.325 GHz.

In order to formulate a detection problem to detect the activity of Starlink downlink signals, a signal model is proposed which solely relies on the periodicity of the transmitted signals. The logic behind the proposed signal model is that in most commercial communication systems, a periodic RS

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is transmitted for synchronization purposes, e.g., primary synchronization signals (PSS) in long-term evolution (LTE) and the fifth generation (5G) signals. The following subsection presents a model for the Starlink LEO SV's downlink signals.

B. Baseband Signal Model

As mentioned previously, in most commercial communication systems, a periodic RS is transmitted, e.g., PSS in OFDMA-based and spreading codes in code division multiple access (CDMA)-based signals. In this paper, the Starlink LEO SV downlink signal is modeled as an unknown periodic signal in the presence of interference and noise. If an RS, such as PSS in OFDMA-based signals, is being periodically transmitted, it will be detected and estimated by the proposed method. The received baseband signal is modeled as

$$r[n] = \alpha c[\tau_n - t_s[n]] \exp(j\theta[\tau_n]) + d[\tau_n - t_s[n]] \exp(j\theta[\tau_n]) + w[n], \quad (1)$$

where $r[n]$ is the received signal at the n th time instant; α is the complex channel gain between the receiver and the Starlink LEO SV; τ_n is the sample time expressed in the receiver time; $c[\tau_n]$ represents the samples of the complex periodic RS with a period of L samples; $t_s[n]$ is the code-delay between the receiver and the Starlink LEO SV at the n th time instant; $\theta[\tau_n] = 2\pi f_D[n]T_s n$ is the carrier phase in radians, where $f_D[n]$ is the instantaneous Doppler frequency at the n th time instant and T_s is the sampling time; $d_i[\tau_n]$ represents the complex samples of some data transmitted from the Starlink LEO SV; and $w[n]$ is measurement noise, which is modeled as a complex, zero-mean, independent, and identically distributed random sequence with variance σ_w^2 .

Starlink LEO SV's signals suffer from very high Doppler shifts. Higher lengths of processing intervals require higher order Doppler models. In order for a Doppler estimation algorithm to provide an accurate estimate of the Doppler frequency, the processing interval should be large enough to accumulate enough power. According to the considered processing interval length in the experiments, it was observed that during the k th processing interval, the instantaneous Doppler frequency is nearly a linear function of time, i.e., $f_D[n] = f_{D_k} + \beta_k n$, where f_{D_k} is referred to as constant Doppler, and β_k is the Doppler rate at the k th processing interval. The coherent processing interval (CPI) is defined as the time interval in which the constant Doppler, f_{D_k} , and the Doppler rate, β_k , are constant.

The received signal at the n th time instant when the Doppler rate is wiped-off is denoted by $r'[n] \triangleq \exp(-j2\pi\beta_k n^2)r[n]$. One can define *the desired RS* which is going to be detected in the acquisition stage as

$$s[n] \triangleq \alpha c[\tau_n - t_s[n]] \exp(j2\pi f_{D_k} T_s n), \quad (2)$$

and the equivalent noise as

$$w_{\text{eq}}[n] = d[\tau_n - t_s[n]] \exp(j2\pi f_{D_k} T_s n) + \exp(-j2\pi\beta_k n^2) w[n]. \quad (3)$$

Hence, $r'[n] = s[n] + w_{\text{eq}}[n]$. Due to the periodicity of the RS, $s[n]$ has the following property

$$s[n + mL] = s[n] \exp(j\omega_k mL) \quad 0 \leq n \leq L - 1, \quad (4)$$

where $\omega_k \triangleq 2\pi f_{D_k} T_s$ is the normalized Doppler at the k th CPI, and $-\frac{1}{2} \leq \omega_k \leq \frac{1}{2}$. A vector of L observation samples corresponding to the m th period of the signal is formed as

$$\mathbf{z}_m \triangleq [r'[mL], r'[mL + 1], \dots, r'[(m + 1)L - 1]]^T. \quad (5)$$

The k th CPI vector is constructed by concatenating M vectors of length L to form the $ML \times 1$ vector

$$\mathbf{y}_k = [\mathbf{z}_{kM}^T, \mathbf{z}_{kM+1}^T, \dots, \mathbf{z}_{(k+1)M-1}^T]^T. \quad (6)$$

Therefore,

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{s} + \mathbf{w}_{\text{eq}_k}, \quad (7)$$

where $\mathbf{s} = [s[1], s[2], \dots, s[L]]^T$, and the $ML \times L$ Doppler matrix is defined as

$$\mathbf{H}_k \triangleq [\mathbf{I}_L, \exp(j\omega_k L) \mathbf{I}_L, \dots, \exp(j\omega_k (M - 1)L) \mathbf{I}_L]^T, \quad (8)$$

where \mathbf{I}_L is an $L \times L$ identity matrix and \mathbf{w}_{eq_k} is the equivalent noise vector.

III. PROPOSED FRAMEWORK

This section, presents the structure of the proposed framework. The proposed receiver consists of two main stages: (i) acquisition and (ii) tracking. In the acquisition stage, an estimate of the period of the RS in the Downlink signal of Starlink SV, and an initial estimate for the Doppler parameters are provided at $k = 0$, which is discussed in the following subsection. In order for the receiver to refine and maintain the Doppler estimate, a tracking stage is also presented.

A. Acquisition

In this section, a detection scheme is proposed to detect the existence of Starlink LEO SVs in the carrier frequency of 11.325 GHz within a bandwidth of 2.5 MHz, at $k = 0$. The following binary hypothesis test is used to detect the Starlink LEO SV signal

$$\begin{cases} \mathcal{H}_0 : \mathbf{y}_0 = \mathbf{w}_{\text{eq}_0} \\ \mathcal{H}_1 : \mathbf{y}_0 = \mathbf{H}_0 \mathbf{s} + \mathbf{w}_{\text{eq}_0}. \end{cases} \quad (9)$$

For a given set of unknown variables $\mathcal{W}_0 = \{L, \omega_0, \beta_0\}$, the generalized likelihood ratio (GLR) detector for the testing hypothesis (9) is known as matched subspace detector [10], [11], and is derived as (see Theorem 9.1 in [12])

$$\mathcal{L}(\mathbf{y}_0 | \mathcal{W}_0) = \frac{\mathbf{y}_0^H \mathbf{P}_{\mathbf{H}_0} \mathbf{y}_0}{\mathbf{y}_0^H \mathbf{P}_{\mathbf{H}_0}^\perp \mathbf{y}_0} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\geq}} \eta, \quad (10)$$

where \mathbf{y}_0^H is the Hermitian transpose of \mathbf{y}_0 , $\mathbf{P}_{\mathbf{H}_0} \triangleq \mathbf{H}_0 (\mathbf{H}_0^H \mathbf{H}_0)^{-1} \mathbf{H}_0^H$ denotes the projection matrix to the column space of \mathbf{H}_0 , $\mathbf{P}_{\mathbf{H}_0}^\perp \triangleq \mathbf{I} - \mathbf{P}_{\mathbf{H}_0}$ denotes the projection matrix onto the space orthogonal to the column space of \mathbf{H}_0 , and η is the threshold which is predetermined according to the probability of false alarm. Since, $\mathbf{H}_k^H \mathbf{H}_k = M \mathbf{I}_L$ for all k , the likelihood $\mathcal{L}(\mathbf{y}_0 | \mathcal{W}_0)$ can be rewritten as $\mathcal{L}(\mathbf{y}_0 | \mathcal{W}_0) = \frac{1}{\frac{\|\mathbf{y}_0\|^2}{M^2 \|\mathbf{H}_0^H \mathbf{y}_0\|^2} - 1} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\geq}} \eta'$, which is a monotonically increasing function of $\frac{\|\mathbf{H}_0^H \mathbf{y}_0\|^2}{\|\mathbf{y}_0\|^2}$. Hence, the GLR detector (10) is equivalent to

$$\frac{\|\mathbf{H}_0^H \mathbf{y}_0\|^2}{\|\mathbf{y}_0\|^2} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\geq}} \eta', \quad (11)$$

where η' is determined according to a desired probability of false alarm. The maximum likelihood estimate of \mathcal{W}_0 is

$$\hat{\mathcal{W}}_0 = \operatorname{argmax}_{L, \omega_0, \beta_0} \|\mathbf{H}_0^H \mathbf{y}_0\|^2. \quad (12)$$

It should be pointed out that the estimated Doppler using (12) results in a constant ambiguity denoted by $\omega_a = 2\pi f_a$. This constant ambiguity is accounted for in the navigation filter.

Fig. 1 demonstrates the likelihood in terms of Doppler frequency and the period for real Starlink downlink signals. The CPI was set to be 200 times the period. As it can be seen in Fig. 1, a Starlink LEO SV downlink signal is detected with a period of 32 μs and at a Doppler frequency of -2745 Hz.

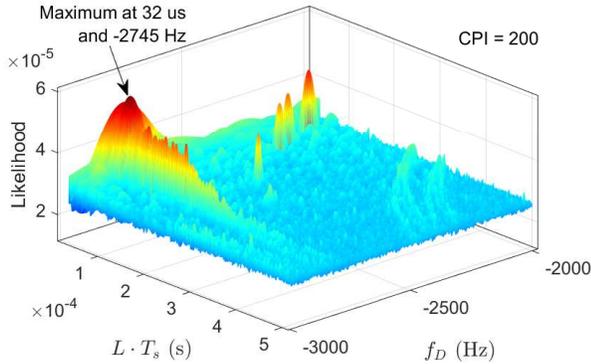


Fig. 1. Acquisition: The likelihood function versus Doppler frequency and the period at Starlink downlink carrier frequency of 11.325 GHz.

B. Doppler Tracking Algorithm

It is important to note that the receiver does not have knowledge of the Doppler ambiguity f_a . The Doppler frequency that will be tracked by the receiver contains this constant ambiguity. In order to track the Doppler, a KF-based tracking loop is developed. The KF formulation allows for arbitrary Doppler model order selection, which is crucial due to the LEO SVs' high-dynamics. The KF-based Doppler tracking algorithm is described below.

1) *Doppler Dynamics Model*: The time-varying component of the continuous-time true Doppler, denoted by $f(t)$, is a function of (i) the true range rate between the LEO SV and the receiver, denoted by $\dot{d}(t)$, and (ii) the time-varying difference between the receiver's and LEO SV's clock bias rate, denoted by $\dot{b}(t)$, expressed in meters per second. Hence, $\omega(t) = 2\pi \left[-\frac{\dot{d}(t)}{\lambda} + \frac{\dot{b}(t)}{\lambda} + f_a \right]$, where, $\omega(t) = 2\pi f(t)$, and λ is the carrier wavelength. The clock bias is assumed to have a constant drift, i.e., $b(t) = a \cdot (t - t_0) + b_0$, where a is the clock drift, b is the constant bias, and t_0 is the initial time. Moreover, simulations with Starlink LEO SVs show that the kinematic model $\ddot{d}(t) = \tilde{w}(t)$, where \tilde{w} is a zero-mean white noise process with power spectral density $q_{\tilde{w}}$ holds for short periods of time. Let k denote the time index corresponding to $t_k = kT + t_0$, where $T = MLT_s$ is the sampling interval also known as subaccumulation period, and ML is the number of subaccumulated samples. The vector $\omega_k \triangleq [\omega_k, \dot{\omega}_k]^T$ is considered as the Doppler state vector for the proposed tracking algorithm. The initial state is given by $\omega_0 = \left[2\pi f_a + \frac{2\pi}{\lambda}(a - \dot{d}(t_0)), -\frac{2\pi}{\lambda}\ddot{d}(t_0) \right]^T$.

2) *KF-Based Doppler Tracking*: Let $\hat{\omega}_{k|l}$ and $\mathbf{P}_{k|l}$ denote the KF estimate of ω_k and corresponding estimation error covariance, respectively, given all measurements up to time-step $l \leq k$. The initial estimate $\hat{\omega}_{0|0}$ with a corresponding $\mathbf{P}_{0|0}$ are provided from the acquisition stage. The KF-based tracking algorithm follows a regular KF for the time-update. The measurement update is discussed next. The KF measurement update equations are carried out based on the maximum likelihood estimate of the Doppler. The Doppler wipe-off is performed as $\tilde{r}_k[i] = r[i + kML] \exp[-j\hat{\theta}_{k+i|k}]$, where $\hat{\theta}_{k+i|k}$ is obtained according to $\hat{\theta}_{k+i|k} = \hat{\omega}_{k|k} i T_s + \hat{\omega}_{k|k} \frac{i^2}{2} T_s^2$, for $i = 0, \dots, ML - 1$. The vector $\tilde{\mathbf{y}}_{k+1}$ is constructed as $\tilde{\mathbf{y}}_{k+1} = [\tilde{r}_k[0], \dots, \tilde{r}_k[ML - 1]]^T$. One can show that (cf. (7))

$$\tilde{\mathbf{y}}_{k+1} = \tilde{\mathbf{H}}_{k+1} \mathbf{s} + \tilde{\mathbf{w}}_{\text{eq},k+1}, \quad (13)$$

where the residual Doppler matrix is

$$\begin{aligned} \tilde{\mathbf{H}}_{k+1} & \triangleq [\mathbf{I}_L, \exp(j\Delta\omega_k L) \mathbf{I}_L, \dots, \exp(j\Delta\omega_{k+1}(M-1)L) \mathbf{I}_L]^T, \end{aligned} \quad (14)$$

and $\Delta\omega_{k+1} = \omega_{k+1} - \hat{\omega}_{k+1|k}$. The proposed KF innovation is given by

$$\nu_{k+1} = \operatorname{argmax}_{\Delta\omega_{k+1}} \frac{1}{M} \|\tilde{\mathbf{H}}_{k+1}^H \tilde{\mathbf{y}}_{k+1}\|^2, \quad (15)$$

which is a direct measure of the Doppler error. The measurement noise is chosen proportional to the Doppler search step size. The initial estimates of the Doppler $\hat{\omega}_{0|0}$ and the Doppler rate $\hat{\omega}_{0|0}$ are obtained from the acquisition stage.

IV. EXPERIMENTAL RESULTS

This section provides the first results for blind Doppler tracking and positioning with Starlink signals of opportunity. A stationary National Instrument (NI) universal software radio peripheral (USRP) 2945R was equipped with a consumer-grade Ku antenna and low-noise block (LNB) downconverter to receive Starlink signals in the Ku-band. The sampling rate was set to 2.5 MHz and the carrier frequency was set to 11.325 GHz, which is one of the Starlink downlink frequencies. The samples of the Ku signal were stored for off-line processing. The tracking results are presented next.

A. Blind Doppler Tracking Results

The USRP was set to record Ku signals over a period of 800 seconds. During this period, a total of six Starlink SVs transmitting at 11.325 GHz passed over the receiver, one at a time. The framework discussed in Section III was used to acquire the downlink signals and track the Doppler frequencies and rates from these LEO SVs, which are shown in Fig. 2 along with the ones predicted from two-line element (TLE) files [3]. It can be seen that the proposed algorithm is tracking the Doppler and the Doppler rate of six Starlink LEO SVs. It can also be seen that the estimated Doppler frequencies have a constant bias compared to the predicted ones from the TLEs.

B. Position Estimation

Next, pseudorange rate observables are formed from the tracked Doppler frequencies by (i) downsampling by a factor D to avoid large time-correlations in the pseudorange

observables and (ii) multiplying by the wavelength to express the Doppler frequencies in meters per second. Let $i \in \{1, 2, 3, 4, 5, 6\}$ denote the SV index. The pseudorange rate observable to the i th SV at time-step $\kappa = k \cdot D$, expressed in meters, is modeled as

$$z_i(\kappa) = \frac{\dot{\mathbf{r}}_{\text{SV}_i}^T(\kappa) [\mathbf{r}_r - \mathbf{r}_{\text{SV}_i}(\kappa)]}{\|\mathbf{r}_r - \mathbf{r}_{\text{SV}_i}(\kappa)\|_2} + a_i + v_{z_i}(\kappa), \quad (16)$$

where \mathbf{r}_r and $\mathbf{r}_{\text{SV}_i}(\kappa)$ are the receiver's and i th Starlink SV three-dimensional (3-D) position vectors, $\dot{\mathbf{r}}_{\text{SV}_i}(\kappa)$ is the i th Starlink SV 2-D velocity vector, a_i is the constant bias due to the unknown Doppler frequency ambiguity f_a , and $v_{z_i}(\kappa)$ is the measurement noise, which is modeled as a zero-mean, white Gaussian random variable with variance $\sigma_i^2(\kappa)$. The value of $\sigma_i^2(\kappa)$ is the first diagonal element of $\mathbf{P}_{\kappa|\kappa}$, expressed in m^2/s^2 . Next, define the parameter vector $\mathbf{x} \triangleq [\mathbf{r}_r^T, a_1, \dots, a_6]^T$. Let \mathbf{z} denote the vector of all the pseudorange observables stacked together, and let \mathbf{v}_z denote the vector of all measurement noises stacked together, which is a zero-mean Gaussian random vector with a diagonal covariance \mathbf{R} whose diagonal elements are given by $\sigma_i^2(\kappa)$. Then, one can readily write the measurement equation given by $\mathbf{z} = \mathbf{g}(\mathbf{x}) + \mathbf{v}_z$, where $\mathbf{g}(\mathbf{x})$ is a vector-valued function that maps the parameter \mathbf{x} to the pseudorange rate observables according to (16). Next, a weighted nonlinear least-squares (WNLS) estimator with weight matrix \mathbf{R}^{-1} is solved to obtain an estimate of \mathbf{x} given by $\hat{\mathbf{x}} = [\hat{\mathbf{r}}_r^T, \hat{a}_1, \dots, \hat{a}_6]^T$. The SV positions were obtained from TLE files and SGP4 software. It is important to note that the TLE epoch time was adjusted for each SV to account for ephemeris errors. This was achieved by minimizing the pseudorange rate residuals for each SV.

Subsequently, the receiver position was estimated using the aforementioned WNLS. The 3-D position error was found to be 22.9 m, while the 2-D position error was 10 m. A skyplot of the Starlink SVs and the environment layout summarizing the positioning results are shown in Fig. 3.

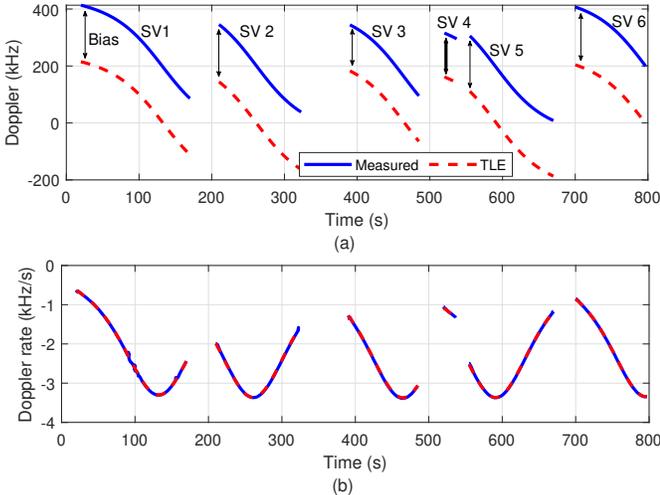


Fig. 2. Experimental results showing measured and predicted (a) Doppler frequencies and (b) Doppler frequency rates from 6 Starlink LEO SVs.

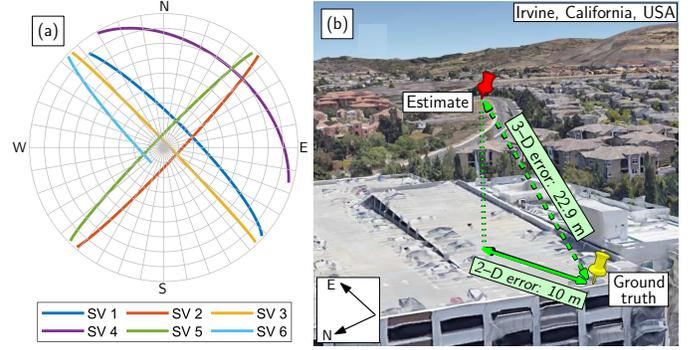


Fig. 3. (a) Skyplot showing the Starlink SVs' trajectories during the experiment. (b) Environment layout and positioning results.

V. CONCLUSION

This letter showed the first Doppler tracking and positioning results with real Starlink LEO SV signals. A model of a Starlink SV's received signal and a detection problem to detect Starlink downlink SV signals were formulated. A KF-based Doppler tracking algorithm was developed to track the Doppler of Starlink downlink signals. Experimental results showed carrier phase tracking of six Starlink LEO SVs over a period of approximately 800 seconds. The experiments also show a 10 m 2-D and 22.9 m 3-D position errors.

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