



Attitude Determination and Control (ADCS)

**16.684 Space Systems Product Development
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ADCS Motivation

- Motivation
 - In order to point and slew optical systems, spacecraft attitude control provides coarse pointing while optics control provides fine pointing
- Spacecraft Control
 - Spacecraft Stabilization
 - Spin Stabilization
 - Gravity Gradient
 - Three-Axis Control
 - Formation Flight
 - Actuators
 - Reaction Wheel Assemblies (RWAs)
 - Control Moment Gyros (CMGs)
 - Magnetic Torque Rods
 - Thrusters
- Sensors: GPS, star sensors, rate gyros, inertial measurement units
- Control Laws
- Spacecraft Slew Manoeuvres
 - Euler Angles
 - Quaternions

Key Question
What are the pointing requirements for satellite pointing?

NEED expendable pointing

- **On-board fuel often determines pointing**
- **Failing gyros are critical**



Outline

- Definitions and Terminology
- Coordinate Systems and Mathematical Attitude Representation
- Rigid Body Dynamics
- Disturbance Torques in Space
- Passive Attitude Control Schemes
- Actuators
- Sensors
- Active Attitude Control Concepts
- ADCS Performance and Stability Measures
- Estimation and Filtering in Attitude Determination
- Maneuvers
- Other System Consideration, Control/Structure interaction
- Technological Trends and Advanced Concepts



Opening Remarks

- Nearly all ADCS Design and Performance can be viewed in terms of RIGID BODY dynamics
- Typically a Major spacecraft system
- For large, light-weight structures with low fundamental frequencies the flexibility needs to be taken into account
- ADCS requirements often drive overall S/C design
- Components are cumbersome, massive and power-consuming
- Field-of-View requirements and specific orientation are critical
- Design, analysis and testing are typically the most challenging of all subsystems with the exception of payload design
- Need a true “systems orientation” to be successful at designing and implementing an ADCS



Terminology

ATTITUDE : Orientation of a defined spacecraft body coordinate system with respect to a defined external frame (GCI, Earth, etc.)

ATTITUDE DETERMINATION: Real-Time or Post-Facto knowledge within a given tolerance, of the spacecraft attitude

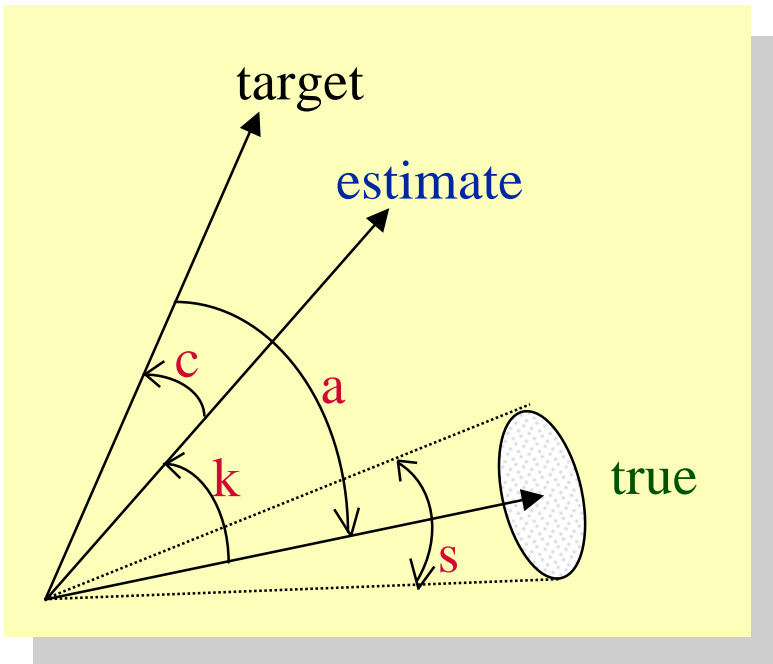
ATTITUDE CONTROL: Maintenance of a desired, specified attitude within a given tolerance

ATTITUDE ERROR: “Low Frequency” spacecraft misalignment usually the intended topic of attitude control

ATTITUDE JITTER: “High Frequency” spacecraft misalignment usually ignored by ADCS; reduced by good design or active pointing/optical control.



Pointing Control Definitions



target	desired pointing direction
true	actual pointing direction
estimate	estimate of true direction
a	pointing accuracy
s	stability (peak-to-peak)
k	knowledge error
c	control error

a = pointing accuracy = attitude error
s = stability = attitude jitter



Attitude Coordinate Systems

(North Celestial Pole)

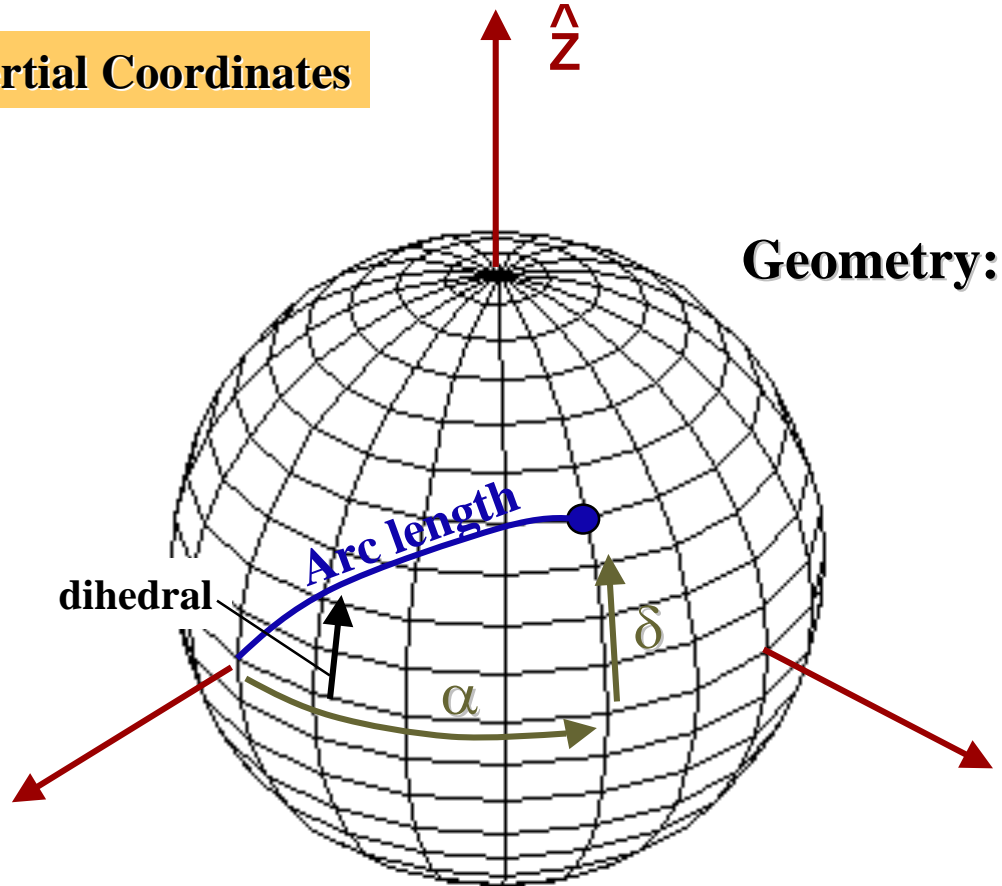
GCI: Geocentric Inertial Coordinates

Cross product

$$\hat{Y} = \hat{Z} \times \hat{X}$$

Geometry: C

**VERNAL
EQUINOX**



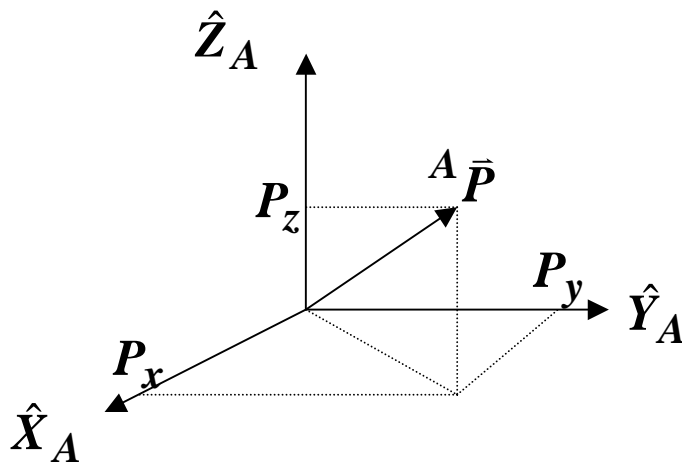
α : **Right Ascension**
 δ : **Declination**

**Inertial Coordinate
System**

X
in the pla



Attitude Description Notations



$\{\cdot\}$ = Coordinate system

\bar{P} = Vector

${}^A\bar{P}$ = Position vector

$${}^A\bar{P} = \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix}$$

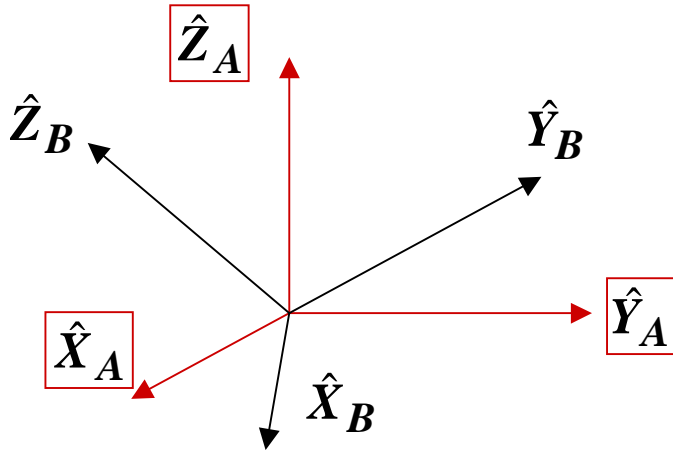
$$\text{Unit vectors of } \{A\} = [\hat{X}_A \ \hat{Y}_A \ \hat{Z}_A] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Describe the orientation of a body:

- (1) Attach a coordinate system to the body
- (2) Describe a coordinate system relative to an inertial reference frame



Rotation Matrix

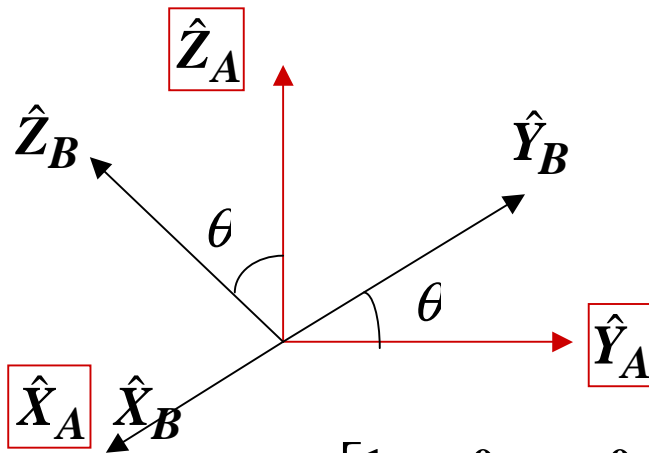


$\{A\}$ = Reference coordinate system

$\{B\}$ = Body coordinate system

Rotation matrix from B to A

$${}^A_B R = \begin{bmatrix} {}^A \hat{X}_B & {}^A \hat{Y}_B & {}^A \hat{Z}_B \\ {}^A \hat{X}_A & {}^A \hat{Y}_A & {}^A \hat{Z}_A \end{bmatrix}$$



$${}^A_B R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

Special properties of rotation matrices

(1) Orthogonal

$$R^T R = I, \quad R^T = R^{-1}$$

(2) Orthonormal

$$\|R\| = 1$$

(3) Not commutative

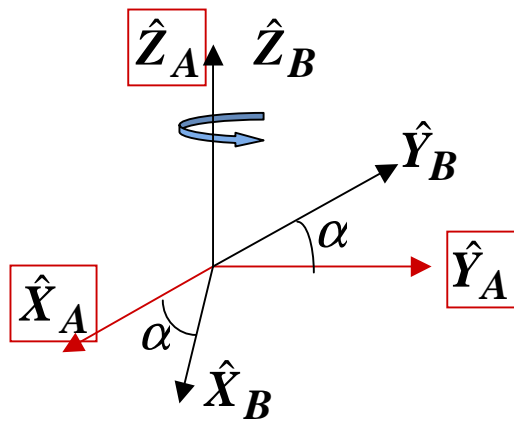
$${}^A_B R \quad {}^B_C R \neq {}^B_C R \quad {}^A_B R$$



Euler Angles (1)

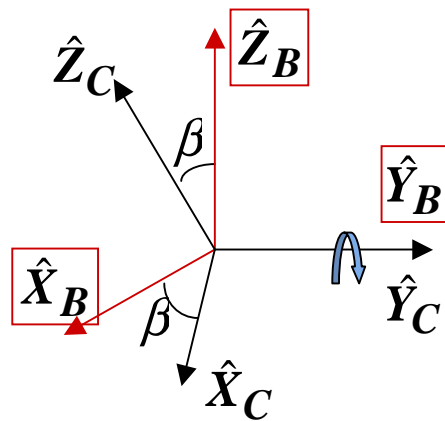
Euler angles describe a sequence of three rotations about axes in order to align one coord. system with a second

Rotate about \hat{Z}_A by α



$${}^A_B R = \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotate about \hat{Y}_B by β



$${}^B_C R = \begin{bmatrix} \cos \beta & 0 & \sin \beta \\ 0 & 1 & 0 \\ -\sin \beta & 0 & \cos \beta \end{bmatrix}$$

Rotate about \hat{Z}_C by γ



$${}^C_D R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \gamma & \sin \gamma \\ 0 & -\sin \gamma & \cos \gamma \end{bmatrix}$$

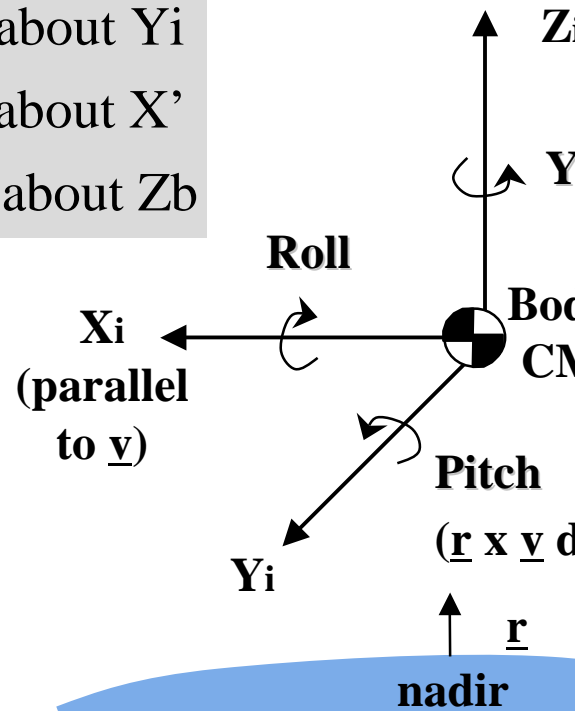
$${}^A_D R = {}^A_B R {}^B_C R {}^C_D R$$



Euler Angles (2)

- Concept used in rotational kinematics to describe body orientation w.r.t. inertial frame
- Sequence of three angles and prescription for rotating one reference frame into another
- Can be defined as a transformation matrix body/inertial as shown: $T_{B/I}$
- Euler angles are non-unique and exact sequence is critical

θ about Y_i
 ϕ about X'
 ψ about Z_b



Goal: Describe kinematics of body frame with respect to rotating inertial frame

$(\text{Pitch, Roll, Yaw}) = (\theta, \phi, \psi) \longrightarrow$

Note:

$$T_{B/I}^{-1} = T_{I/B} = T_{B/I}^T$$

Transformation from Body to “Inertial” frame:

$$T_{B/I} = \underbrace{\begin{bmatrix} \cos \psi & \sin \psi & 0 \\ -\sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{YAW}} \cdot \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & \sin \phi \\ 0 & -\sin \phi & \cos \phi \end{bmatrix}}_{\text{ROLL}} \cdot \underbrace{\begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{Pitch}}$$



Quaternions

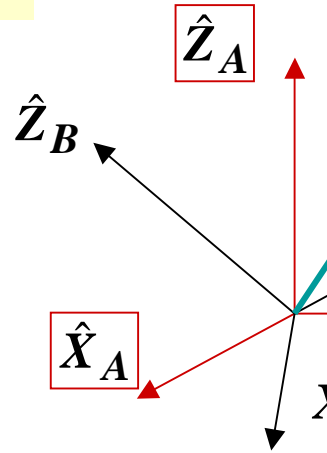
- Main problem computationally is the existence of a singularity
- Problem can be avoided by an application of Euler's theorem:

$$Q = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \begin{bmatrix} \bar{q} \\ q_4 \end{bmatrix}$$

\bar{q} = A vector
axis of rotation
 q_4 = A scalar
amount of rotation

EULER'S THEOREM

The Orientation of a body is uniquely specified by a vector giving the direction of a body axis and a scalar specifying a rotation angle about the axis.



- Definition introduces a redundant fourth element, which eliminates the singularity.
- This is the “quaternion” concept
- Quaternions have no intuitively interpretable meaning to the human mind, but are computationally convenient

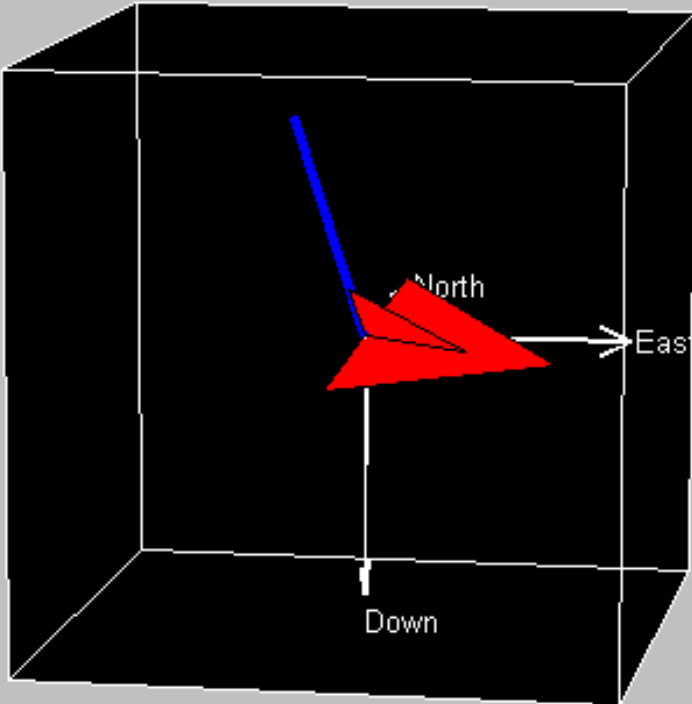
A: Inertial
B: Body

$${}^A \hat{K} = \begin{bmatrix} k_x \\ k_y \\ k_z \end{bmatrix}$$



Quaternion Demo (MATLAB)

Quaternion Demonstration



Fast Render

 Dynamic Static

Euler Angles

Yaw Deg
-180 ◀ ▶ 180

Pitch Deg
-90 ◀ ▶ 90

Roll Deg
-180 ◀ ▶ 180

Quaternion Representation

$Q = [-0.17 \ -0.22 \ -0.8 \ -0.53]$

Azimuth Deg
-360 ◀ ▶ 360

Elevation Deg
-180 ◀ ▶ 180

Beta Deg
0 ◀ ▶ 360



Comparison of Attitude Descriptions

Method	Euler Angles	Direction Cosines	Angular Velocity ω	Quaternion
Pluses	If given ϕ, ψ, θ then a unique orientation is defined	Orientation defines a unique dir-cos matrix \mathbf{R}	Vector properties, commutes w.r.t addition	Commutative, robust, Ideal for control
Minuses	Given orient then Euler non-unique Singularity	6 constraints must be met, non-intuitive	Integration w.r.t time does not give orientation Needs transform	Not commutative, Needs initial condition

Best for analytical and ACS design work

Must store initial condition

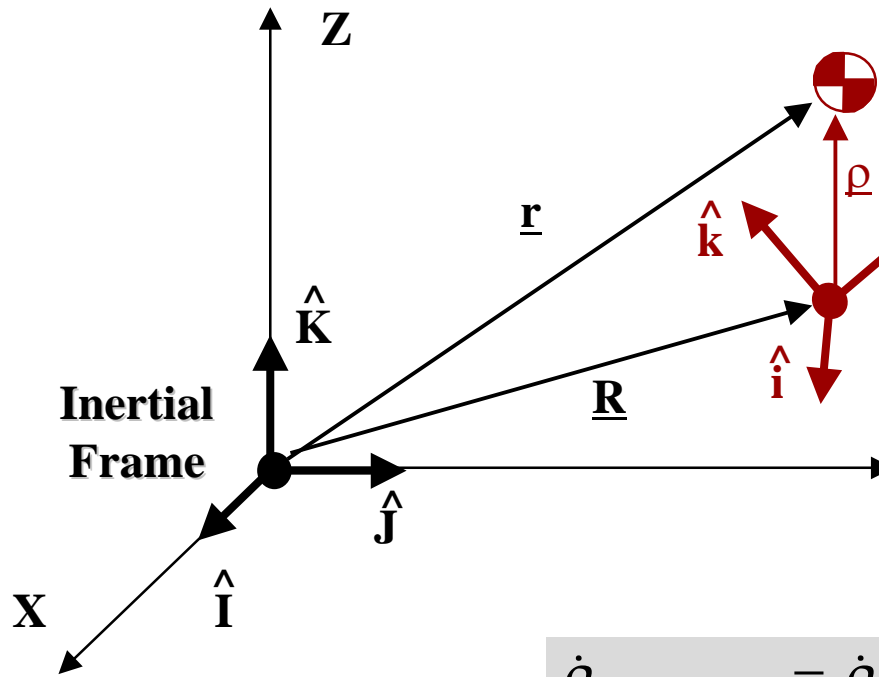
Best for digital implementation



Rigid Body Kinematics

Time Derivatives:
(non-inertial)

$\underline{\omega}$ = Angular velocity
of Body Frame



Applied to
position vector \underline{r} :

BASIC RULE:

$$\dot{\underline{\rho}}_{\text{INERTIAL}} = \dot{\underline{\rho}}$$

$$\underline{r} = \underline{R} + \underline{\rho} \quad \text{Position}$$

Expressed in
the Inertial Frame

$$\dot{\underline{r}} = \dot{\underline{R}} + \dot{\underline{\rho}}_{\text{BODY}} + \underline{\omega} \times \underline{\rho} \quad \text{Rate}$$

$$\ddot{\underline{r}} = \ddot{\underline{R}} + \ddot{\underline{\rho}}_{\text{BODY}} + 2\underline{\omega} \times \dot{\underline{\rho}}_{\text{BODY}} + \dot{\underline{\omega}} \times \underline{\rho} + \underline{\omega} \times (\underline{\omega} \times \underline{\rho})$$

↑ Inertial accel of CM
↑ relative accel w.r.t. CM
↑ coriolis
↑ angular accel
↑ centripetal

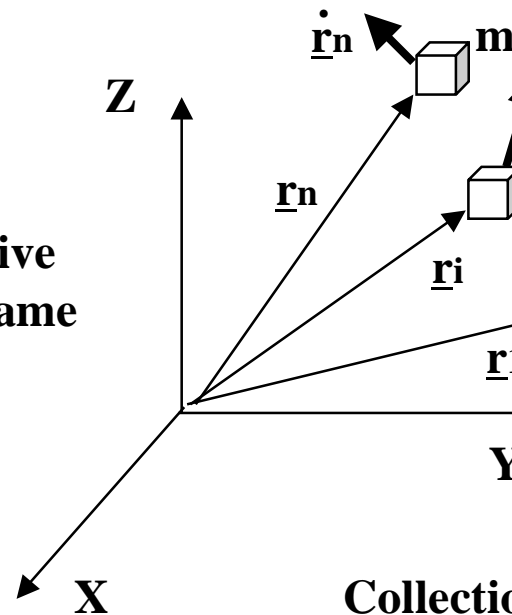


Angular Momentum (I)

Angular Momentum

$$\underline{H}_{\text{total}} = \sum_{i=1}^n \underline{r}_i \quad m_i \underline{\dot{r}}_i$$

System in motion relative to Inertial Frame



If we assume that

- (a) **Origin of Rotating Frame in Body CM**
- (b) **Fixed Position Vectors \underline{r}_i in Body Frame (Rigid Body)**

Angular Momentum Decomposition

Then :

$$\underline{H}_{\text{total}} = \underbrace{\left(\sum_{i=1}^n m_i \right) \underline{R} \quad \underline{\dot{R}}}_{\text{ANGULAR MOMENTUM OF TOTAL MASS W.R.T INERTIAL ORIGIN}} + \underbrace{\sum_{i=1}^n m_i \underline{\rho}_i \quad \underline{\dot{\rho}}_i}_{\square \underline{H}_{\text{BODY}}}$$

ANGULAR MOMENTUM OF TOTAL MASS W.R.T INERTIAL ORIGIN

$\square \underline{H}_{\text{BODY}}$
BODY ANGULAR MOMENTUM ABOUT CENTER OF MASS

Not meas iner



Angular Momentum (II)

For a RIGID BODY
we can write:

$$\underline{\dot{\rho}}_i = \underbrace{\underline{\dot{\rho}}_{i,BODY}}_{\text{RELATIVE MOTION IN BODY}} + \underline{\omega} \times \underline{\rho}_i =$$

And we are able to write:

$$\underline{H} = I \underline{\omega}$$

RIGID BODY, CM COORDINATES
 \underline{H} and $\underline{\omega}$ are resolved in the body frame

“The vector of angular momentum in the body frame is the product of the 3x3 Inertia matrix and the 3x1 vector of angular velocity”

Inertia Matrix
Properties:

Real Symmetric ; 3x3 Tensor ; coordinate dependent

$$I = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{bmatrix}$$

$$I_{11} = \sum_{i=1}^n m_i (\rho_{i2}^2 + \rho_{i3}^2) \quad I_{12} = I_{21} =$$

$$I_{22} = \sum_{i=1}^n m_i (\rho_{i1}^2 + \rho_{i3}^2) \quad I_{13} = I_{31} =$$

$$I_{33} = \sum_{i=1}^n m_i (\rho_{i1}^2 + \rho_{i2}^2) \quad I_{23} = I_{32} =$$



Kinetic Energy and Euler Equations

Kinetic Energy

$$E_{\text{total}} = \underbrace{\frac{1}{2} \left(\sum_{i=1}^n m_i \right) \dot{R}^2}_{\text{E-TRANS}} + \underbrace{\frac{1}{2} \sum_{i=1}^n m_i \dot{\rho}_i^2}_{\text{E-ROT}}$$

For a RIGID BODY, CM Coordinates with $\underline{\omega}$ resolved in body axis frame

$$E_{\text{ROT}} = \frac{1}{2} \underline{\omega} \cdot \underline{H} = \frac{1}{2}$$

$$\underline{\dot{H}} = \underline{T} - \underline{\omega} \left[\underline{I} \underline{\omega} \right]$$

Sum of external and internal

In a BODY-FIXED, PRINCIPAL AXES CM FRAME: Euler E

$$\dot{H}_1 = I_1 \dot{\omega}_1 = T_1 + (I_{22} - I_{33}) \omega_2 \omega_3$$

$$\dot{H}_2 = I_2 \dot{\omega}_2 = T_2 + (I_{33} - I_{11}) \omega_3 \omega_1$$

$$\dot{H}_3 = I_3 \dot{\omega}_3 = T_3 + (I_{11} - I_{22}) \omega_1 \omega_2$$

No general solution
Particular solutions
simple torque
simulation usu



Torque Free Solutions of Euler's Eq.

TORQUE-FREE CASE:

An important special case is the torque-free motion of a symmetric body spinning primarily about its symmetrical axis.

By these assumptions:

$$\omega_x, \omega_y \ll \omega_z = \Omega \quad I_{xx} = I_{yy} < I_{zz}$$

The components of angular velocity then become:

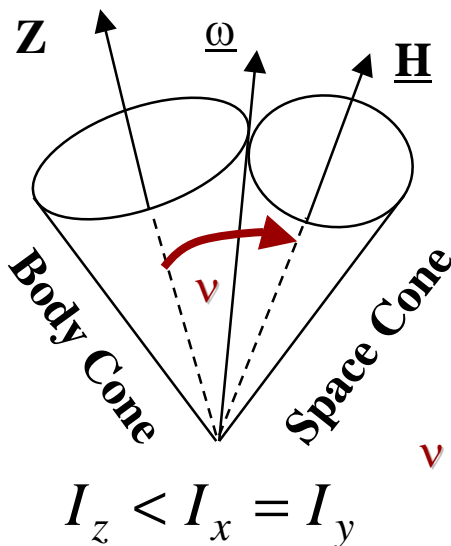
$$\omega_x(t) = \omega_{x0} \cos \omega_n t$$

$$\omega_y(t) = \omega_{y0} \cos \omega_n t$$

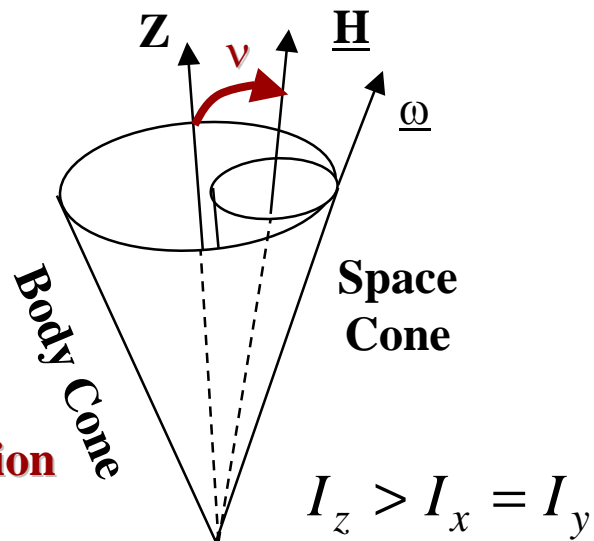
And the Euler equations become:

The ω_n is defined as the "natural" or "nutation" frequency of the body:

$$\omega_n^2 = K_x K_y \Omega^2$$



ν : nutation angle



$$\dot{\omega}_x = -\omega_n \omega_y$$

$$\dot{\omega}_y = \omega_n \omega_x$$

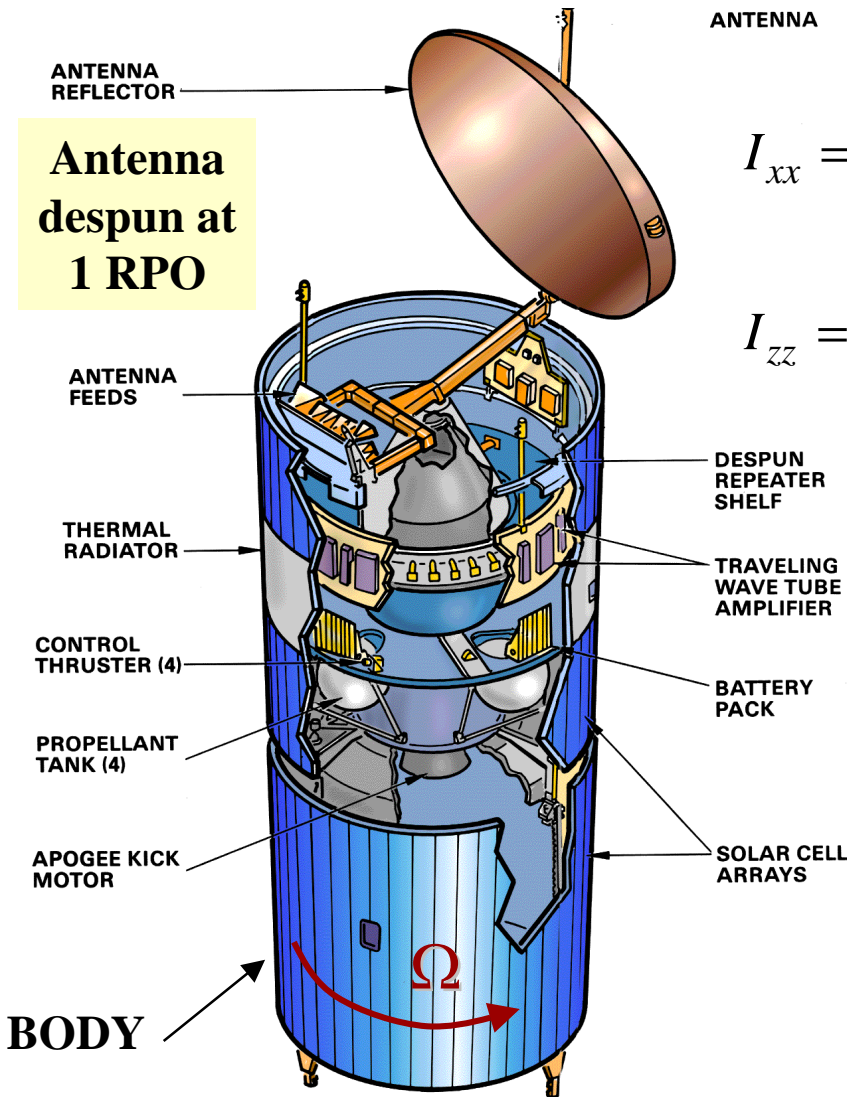
$$\dot{\omega}_z = 0$$

\underline{H} and $\underline{\omega}$ unless a priori



Spin Stabilized Spacecraft

UTILIZED TO STABILIZE SPINNERS

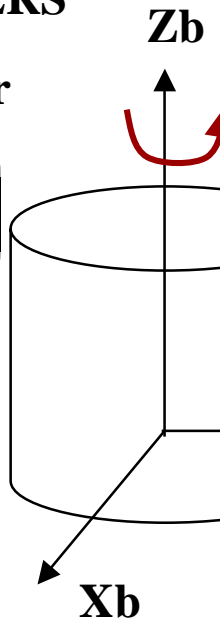


Antenna despun at 1 RPO

Perfect Cylinder

$$I_{xx} = I_{yy} = \frac{m}{4} \left(\frac{L^2}{3} + R^2 \right)$$

$$I_{zz} = \frac{mR^2}{2}$$



DUAL SPIN

- Two bodies rotating about a common axis
- Behaves like simple spin is despun (antennas, solar panels)
- requires torquers (jets) momentum control and dampers for stability
- allows relaxation of m

HS 376 SPACECRAFT CONFIGURATION



Disturbance Torques

Assessment of expected disturbance torques is an essential part of rigorous spacecraft attitude control design.

Typical Disturbances

- Gravity Gradient: “Tidal” Force due to $1/r^2$ gravitational force for long, extended bodies (e.g. Space Shuttle, Tethered vehicles).
- Aerodynamic Drag: “Weathervane” Effect due to an offset between the CM and the drag center of Pressure (CP). Only a factor in Earth orbit.
- Magnetic Torques: Induced by residual magnetic moment of spacecraft as a magnetic dipole. Only within magnetosphere.
- Solar Radiation: Torques induced by CM and solar CP of spacecraft. Can be compensated with differential reflectivity or reaction wheels.
- Mass Expulsion: Torques induced by leaks or jettisoned components.
- Internal: On-board Equipment (machinery, wheels, cryocoolers, etc...). No net effect, but internal momentum exchange affects attitude.



Gravity Gradient

$$n = \sqrt{\mu / a^3} = \text{ORB}$$

Gravity Gradient:

- 1) \perp Local vertical
- 2) 0 for symmetric spacecraft
- 3) proportional to $\propto 1/r^3$

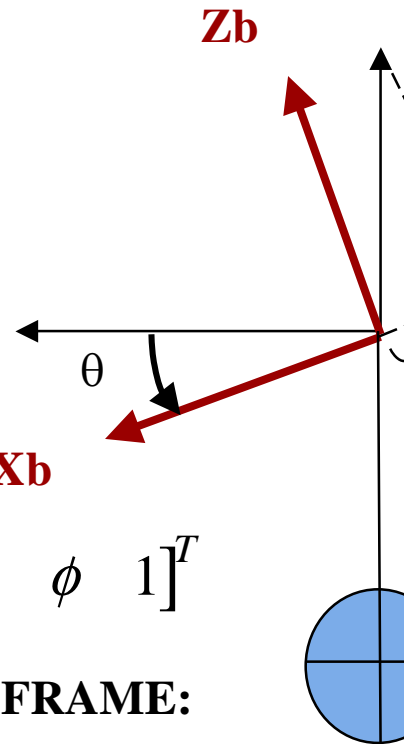
Gravity Gradient Torques

$$\underline{T} = 3n^2 \cdot \hat{r} \left[\underline{I} \hat{r} \right]$$

In Body Frame

Small angle approximation

$$\hat{r} = \begin{bmatrix} -\sin \theta & \sin \phi & 1 - \sin^2 \theta - \sin^2 \phi \end{bmatrix}^T \cong \begin{bmatrix} -\theta & \phi & 1 \end{bmatrix}^T$$



Resulting torque in BODY FRAME:

Typical Values:

$$\begin{aligned} I &= 1000 \text{ kgm}^2 \\ n &= 0.001 \text{ s}^{-1} \\ T &= 6.7 \times 10^{-5} \text{ Nm/deg} \end{aligned}$$

$$\therefore T \cong 3n^2 \begin{bmatrix} (I_{zz} - I_{yy})\phi \\ (I_{zz} - I_{xx})\theta \\ 0 \end{bmatrix}$$

Pitch

$$\omega_{lib} = n$$



Aerodynamic Torque

$$\underline{T} = \underline{r} \times \underline{F}_a$$

\underline{r} = Vector from body CM
to Aerodynamic CP

$$F_a = \frac{1}{2} \rho V^2 S C_D$$

\underline{F}_a = Aerodynamic Drag Vector
in Body coordinates

Aerodynamic
Drag Coefficient

$$1 \leq C_D \leq 2$$

Typically in this Range
Free Molecular Flow

S = Frontal projected Area

V = Orbital Velocity

ρ = Atmospheric Density

Typical Values:

$$C_d = 2.0$$

$$S = 5 \text{ m}^2$$

$$r = 0.1 \text{ m}$$

$$\rho = 4 \times 10^{-12} \text{ kg/m}^3$$

$$T = 1.2 \times 10^{-4} \text{ Nm}$$

Notes

(1) \underline{r} varies with Attitude

(2) ρ varies by factor of 5-10 at
a given altitude

(3) C_D is uncertain by 50 %

$$2 \times 10^{-9} \text{ kg/m}^3$$

$$3 \times 10^{-10} \text{ kg/m}^3$$

$$7 \times 10^{-11} \text{ kg/m}^3$$

$$4 \times 10^{-12} \text{ kg/m}^3$$

Exponential I

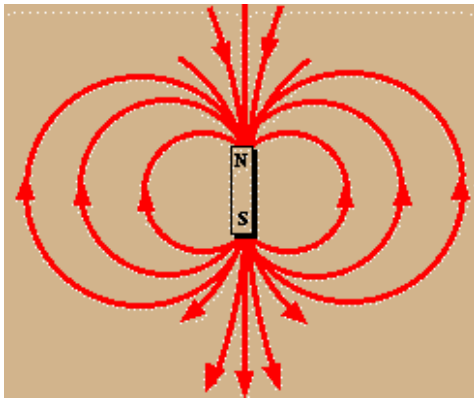


Magnetic Torque

$$\underline{T} = \underline{M} \times \underline{B}$$

M = Spacecraft residual dipole
in AMPERE-TURN-m2 (SI)
or POLE-CM (CGS)

M = is due to current loops and
residual magnetization, and will
be on the order of 100 POLE-CM
or more for small spacecraft.



Typical Values:

$$B = 3 \times 10^{-5} \text{ TESLA}$$

$$M = 0.1 \text{ Atm}^2$$

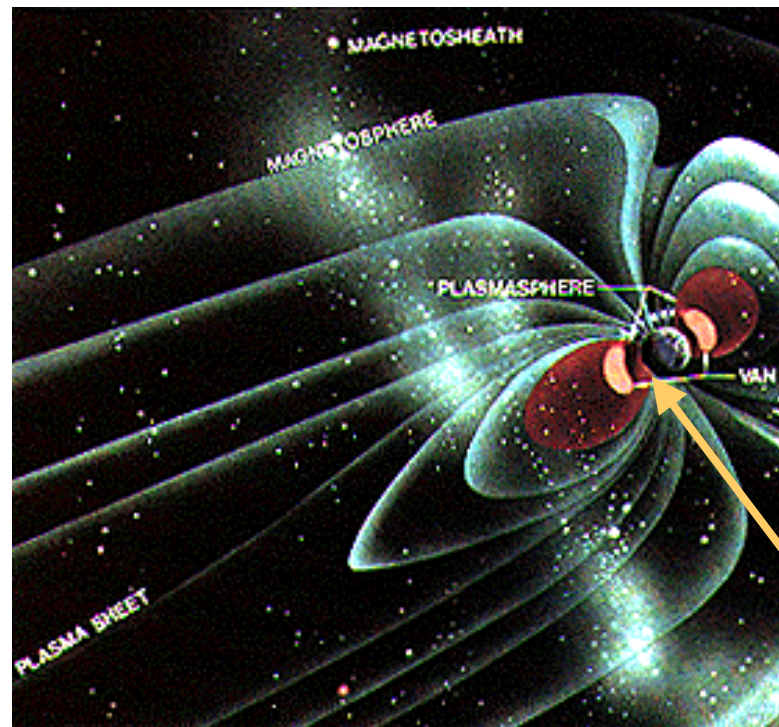
$$T = 3 \times 10^{-6} \text{ Nm}$$

B = Earth magnetic field vector
spacecraft coordinates (BODY FR)
in TESLA (SI) or Gauss (CGS) u

B varies as $1/r^3$, with its direction
along local magnetic field line

Conversions:

$$1 \text{ Atm}^2 = 1000 \text{ POLE-CM} , 1 \text{ TESLA}$$





Solar Radiation Torque

$$\underline{T} = \underline{r} \times \underline{F}_s$$

$$F_s = (1 + K) P_s S$$

$$P_s = I_s / c$$

$$I_s = 1400 \text{ W/m}^2 \quad @ \quad 1 \text{ A.U.}$$

Notes:

- (a) Torque is always \perp to sun line
- (b) Independent of position or velocity as long as in sunlight

Typical Values:

$$\begin{aligned} K &= 0.5 \\ S &= 5 \text{ m}^2 \\ r &= 0.1 \text{ m} \\ T &= 3.5 \times 10^{-6} \text{ Nm} \end{aligned}$$

\underline{r} = Vector from Body CM to optical Center-of-Pressure (COP)

\underline{F}_s = Solar Radiation pressure in BODY FRAME coordinates

K = Reflectivity, $0 < K < 2$

S = Frontal Area

I_s = Solar constant, dependent on heliocentric altitude

SUN

Significant for spacecraft with large frontal area (e.g. NGST)





Mass Expulsion and Internal Torques

Mass Expulsion Torque: $\underline{T} = \underline{r} \times \underline{F}$

Notes:

- (1) **May be deliberate (Jets, Gas venting) or accidental**
- (2) **Wide Range of r, F possible; torques can dominate**
- (3) **Also due to jettisoning of parts (covers, cannisters)**

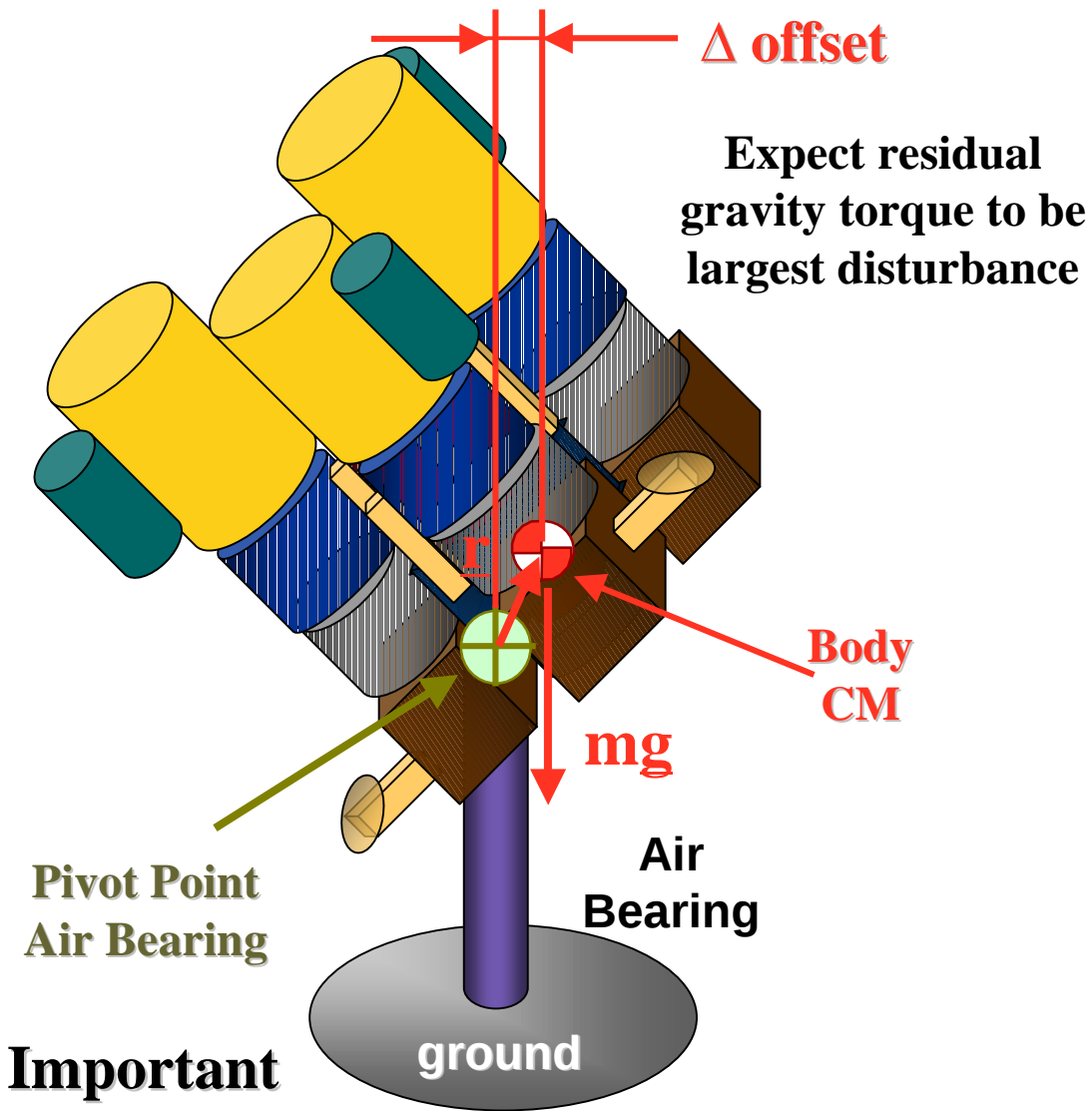
Internal Torque:

Notes:

- (1) **Momentum exchange between moving parts has no effect on System H, but will affect attitude control loops**
- (2) **Typically due to antenna, solar array motion or to deployable booms and**



Disturbance Torque for CDIO



Important to balance precisely !

Initial Assumption:

$$|T| = |\underline{r} \quad m\underline{g}| \cong 0.001 \cdot 100 \cdot$$



Passive Attitude Control (1)

Passive control techniques take advantage of basic physics principles and/or naturally occurring forces by designing the spacecraft so as to enhance the effect of one force while reducing the effect of others.

SPIN STABILIZED

- Requires Stable Inertia Ratio: $I_z > I_y = I_x$
- Requires Nutation damper: Eddy Current, Ball-in-Tube, Viscous Ring, Active Damping
- Requires Torquers to control precession (spin axis drift) magnetically or with jets
- Inertially oriented

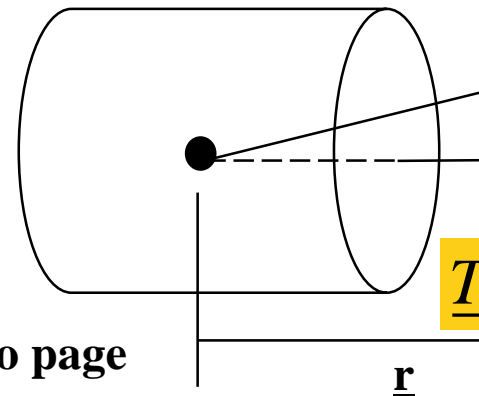
$$\dot{H} =$$

$$\dot{H} =$$

$$\therefore \Delta$$

$$\Delta H = 2H \sin \frac{\Delta \theta}{2} \cong H \Delta \theta = I \omega \cdot \Delta \theta$$

Precession:



\underline{F} into page

\underline{T}

\underline{r}

Large ω
=
gyroscopic
stability

$$\Delta \theta \cong \frac{rF \Delta t}{H} = \frac{rF}{I \omega} \Delta t$$

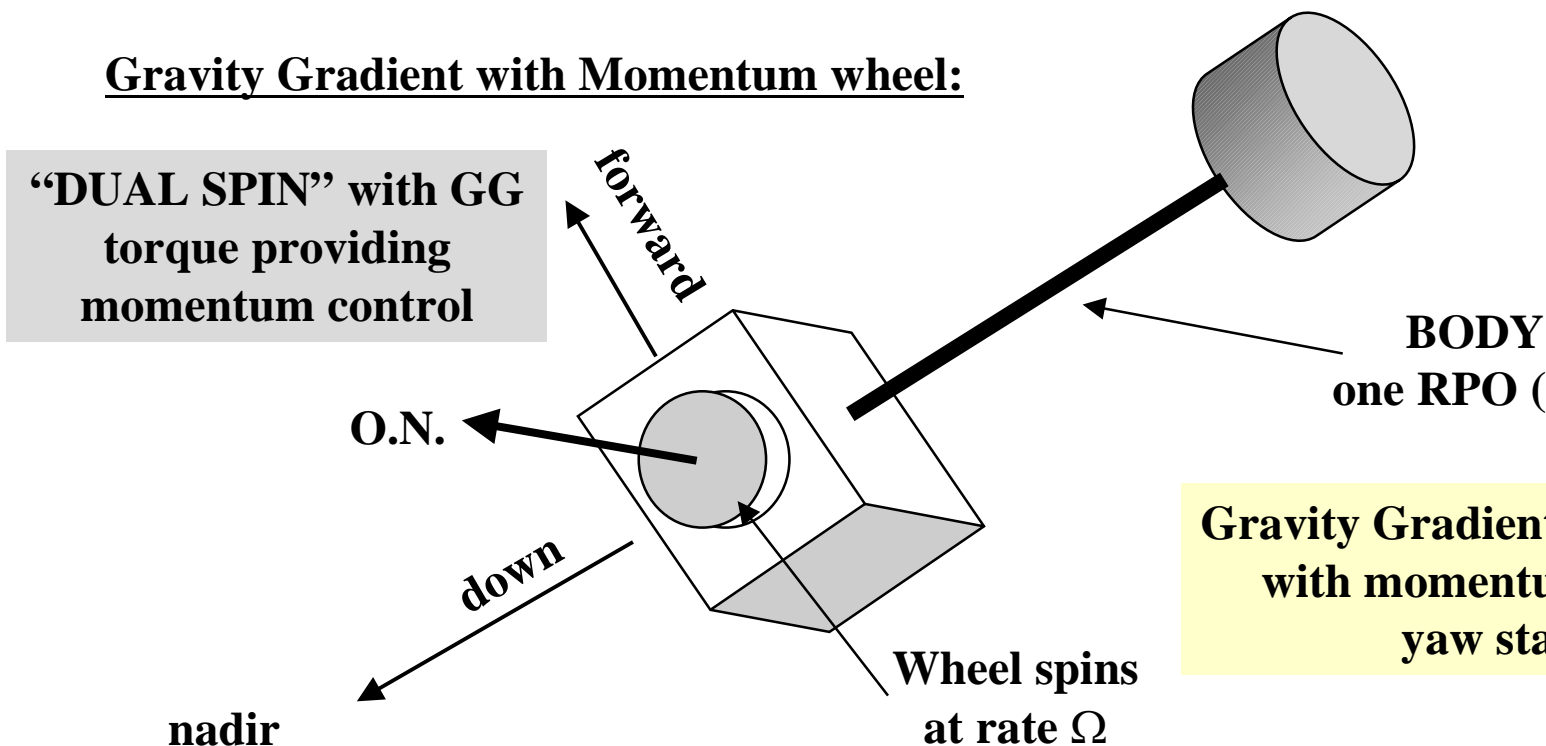


Passive Attitude Control (2)

GRAVITY GRADIENT

- Requires stable Inertias: $I_z \ll I_x, I_y$
- Requires Libration Damper: Eddy Current Hysteresis Rods
- Requires no Torquers
- Earth oriented
- No Yaw Stability (can add momentum)

Gravity Gradient with Momentum wheel:





Active Attitude Control

Active Control Systems directly sense spacecraft attitude and supply a torque command to alter it as required. This is the basic concept of feedback control.

- Reaction Wheels most common actuator
- Fast; continuous feedback control
- Moving Parts
- Internal Torque only; external still required for “momentum dumping”
- Relatively high power, weight, cost
- Control logic simple for independent axes (can get complicated with redundancy)

Typical Reaction (Momentum) Wheel Data:

Operating Range
Angular Momentum
1.3 M
Angular Momentum
4.0 M
Reaction Torque



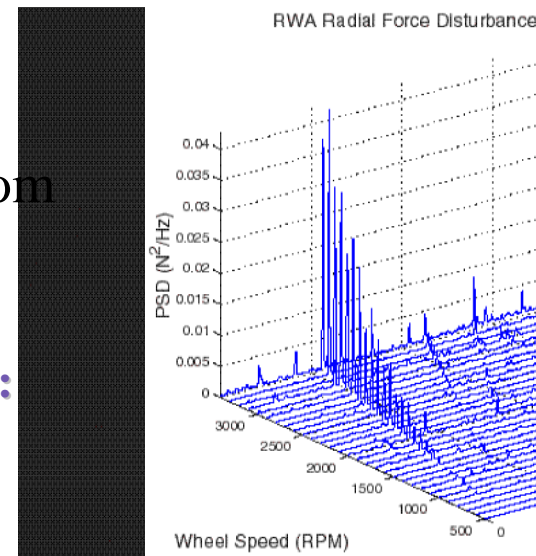
Actuators: Reaction Wheels

- One creates torques on a spacecraft by creating equal but opposite torques on **Reaction Wheels** (flywheels on motors).
 - For three-axes of torque, three wheels are necessary. Usually four wheels for redundancy (use wheel speed biasing equation)
 - If external torques exist, wheels will angularly accelerate to counter these torques. They will eventually reach an RPM limit (~ 6000 RPM) at which time they must be desaturated.
 - Static & dynamic imbalances can induce vibrations (mounting)
 - Usually operate around some nominal spin rate to avoid stiction



Ithaco RWA's
(www.ithaco.com/products.html)

Waterfall plot:



Needs to be carefully balanced !



Actuators: Magnetic Torquers

Magnetic Torquers

- Often used for Low Earth Orbit (LEO) satellites
- Useful for initial acquisition maneuvers
- Commonly use for momentum desaturation (“dumping”) in reaction wheel systems
- May cause harmful influence on star trackers
- Can be used
 - for attitude control
 - to de-saturate reaction wheels
- Torque Rods and Coils
 - Torque rods are long thin rods
 - Use current to generate magnetic field
 - This field will try to align with Earth’s magnetic field creating a torque on the satellite
 - Can also be used to control orbital location as well as orbital location



ACS Actuators: Jets / Thrusters

- Thrusters / Jets
 - Thrust can be used to control attitude but at the cost of consuming fuel
 - Calculate required fuel using “Rocket Equation”
 - Advances in micro-propulsion make this approach more feasible. Typically want $I_{sp} > 1000$ sec
- Use consumables such as Freon, N₂ or Hydrazine
- Must be ON/OFF operation. Proportional control usually not feasible: pulse width modulation (PWM)
- Redundancy usually required. Makes the system more complex and expensive
- Fast, powerful
- Often introduces attitude coupling
- Standard equipment on spacecraft
- May be used to “unload” angular momentum on uncontrolled spacecraft.



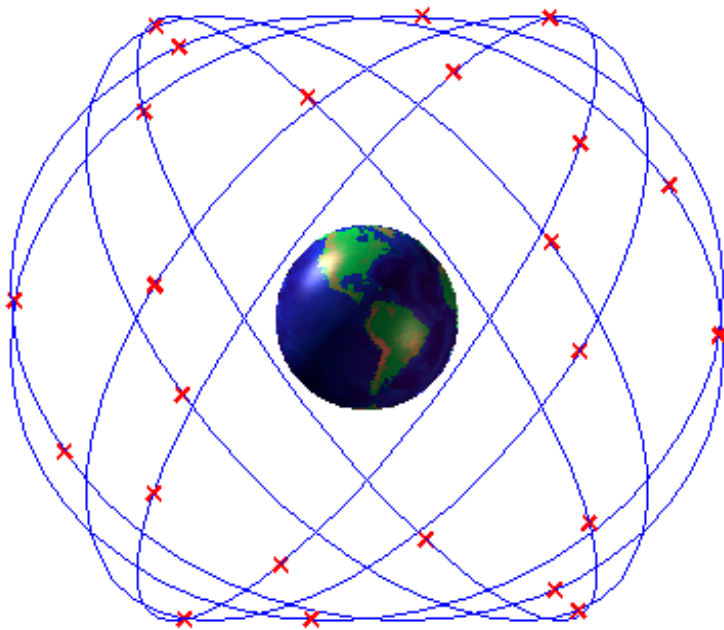
ACS Sensors: GPS and Magnetometers

Global Positioning System (GPS)

- Currently 27 Satellites
- 12hr Orbits
- Accurate Ephemeris
- Accurate Timing
 - Stand-Alone 100m
 - DGPS 5m
 - Carrier-smoothed DGPS 1-2m

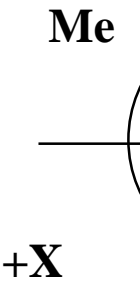
Magnetometers

- Measure component of ambient magnetic field
- Sensitive to field from electronics, mounted
- Get attitude information by comparing measured field to tilted dipole model



$$\begin{bmatrix} B_{north} \\ B_{east} \\ B_{down} \end{bmatrix} = \left(\frac{6378}{r_{km}} \right)^3 \begin{bmatrix} -C_\phi & S_\phi C_\lambda \\ 0 & S_\lambda \\ -2S_\phi & -2C_\phi C_\lambda \end{bmatrix}$$

Where: C=cos , S=sin, ϕ =latitude
Units: nTesla





ACS Sensors: Rate Gyros and IMUs

○ Rate Gyros (Gyroscopes)

- Measure the angular rate of a spacecraft relative to inertial space
- Need at least three. Usually use more for redundancy.
- Can integrate to get angle.

However,

- DC bias errors in electronics will cause the output of the integrator to ramp and eventually saturate (drift)
- Thus, need inertial update



- Mechanical gyros (accurate, heavy)
- Ring Laser (RLG)
- MEMS-gyros

○ Inertial Measurement

- Integrated unit with mounting hardware and software
- measure rotation of rate gyros
- measure translation with accelerometers
- often mounted on platform (fixed in)
- Performance 1: gyro (range: 0 .003 deg)
- Performance 2: lin to $5E-06$ g/g² ov
- Typically frequen external measure Trackers, Sun sen Kalman Filter

Courtesy of Silicon Sensing Systems, Ltd. Used with permission.

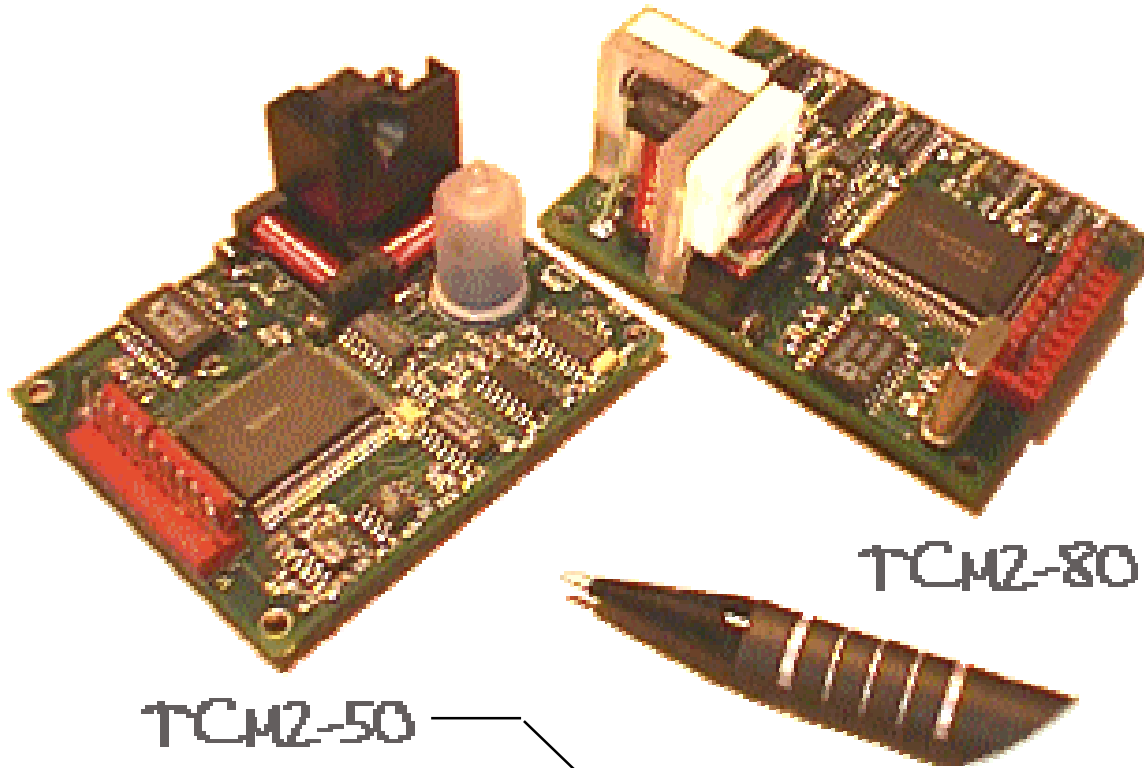


ACS Sensor Performance Summary

Reference	Typical Accuracy	Remarks
Sun	1 min	Simple, reliable, low cost, not always visible
Earth	0.1 deg	Orbit dependent; usually requires scan; relatively expensive
Magnetic Field	1 deg	Economical; orbit dependent; low altitude only; low accuracy
Stars	0.001 deg	Heavy, complex, expensive, most accurate
Inertial Space	0.01 deg/hour	Rate only; good short term reference; can be heavy, power, cost



CDIO Attitude Sensing



Will not be use/afford STAR

From where an attitude for inertial

Potential Electronic Magnetometer Tilt Sens

Specifications:

Heading accuracy: +/- 1.0 deg RMS @ +/- 20 deg tilt
Resolution 0.1 deg, repeatability: +/- 0.3 deg
Tilt accuracy: +/- 0.4 deg, Resolution 0.3 deg
Sampling rate: 1-30 Hz

Problem: Accuracy insufficient to meet requirements a will need FINE POINTING mode



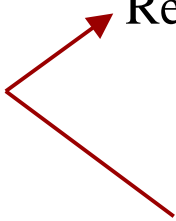
Spacecraft Attitude Schemes

- Spin Stabilized Satellites
 - Spin the satellite to give it gyroscopic stability in inertial space
 - Body mount the solar arrays to guarantee partial illumination by sun at all times
 - EX: early communication satellites, stabilization for orbit changes
 - Torques are applied to precess the angular momentum vector
- De-Spun Stages
 - Some sensor and antenna systems require inertial or Earth referenced pointing
 - Place on de-spun stage
 - EX: Galileo instrument platform
- Gravity Gradient Stabilization
 - “Long” satellites stabilize towards Earth since they feel slightly more gravity force.
 - Good for Earth-referenced pointing
 - EX: Shuttle gravity gradient stabilization minimizes ACS thruster use
- Three-Axis Stabilization
 - For inertial or Earth referenced pointing
 - Requires active control
 - EX: Modern communication satellites, International Space Station, MIR, Hubble Space Telescope



ADCS Performance Comparison

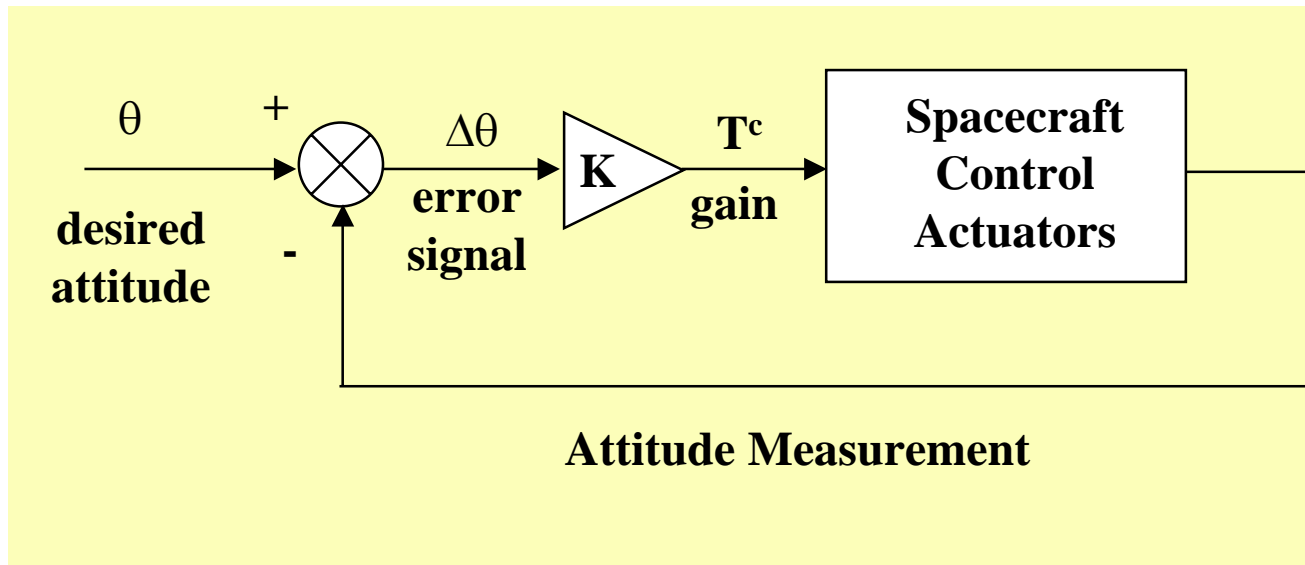
Method	Typical Accuracy	Remarks
Spin Stabilized	0.1 deg	Passive, simple; s inertial, low cost, rings
Gravity Gradient	1-3 deg	Passive, simple; c body oriented; low
Jets	0.1 deg	Consumables requ high cost
Magnetic	1 deg	Near Earth; slow weight, low cost
Reaction Wheels	0.01 deg	Internal torque; re other momentum high power, cost



3-axis stabilized, active control most common choice for precision



ACS Block Diagram (1)



Feedback Control Concept:

$$T^c = K \cdot \Delta\theta$$

Corrected
torque

Force or torque is proportional to deflection. This is the equation, which governs a simple linear or rotational “spring” system. If the spacecraft responds “quickly we can estimate the required gain and system bandwidth.



Gain and Bandwidth

Assume control saturation half-width θ_{sat} at torque command T

$$K \cong \frac{T_{sat}}{\theta_{sat}} \quad \text{hence} \quad \ddot{\theta} + \left(\frac{K}{I} \right) \theta_{sat} \cong 0$$

Recall the oscillator frequency of a simple linear, torsional spring:

$$\omega = \sqrt{\frac{K}{I}} \quad [\text{rad/sec}] \quad \text{I = moment of inertia}$$

This natural frequency is approximately equal to the system bandwidth. Also,

$$f = \frac{\omega}{2\pi} \quad [\text{Hz}] \quad \Rightarrow \quad \tau = \frac{1}{f} = \frac{2\pi}{\omega}$$

Is approximately the system time constant τ .

Note: we can choose any two of the set:

$$\ddot{\theta}, \theta_{sat}, \omega$$

EXAM

$$\theta_{sat} = 10^{-2}$$

$$T_{sat} = 10$$

$$I = 1000$$

$$\therefore K = 1000$$

$$\omega = 1$$

$$f = 0.16$$

$$\tau = 6.3$$



Feedback Control Example

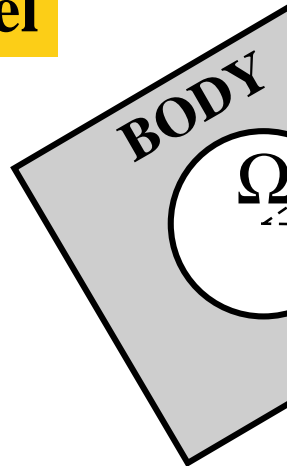
Pitch Control with a single reaction wheel

Rigid Body Dynamics

$$I\ddot{\theta} = T_w + T_{ext} = I\dot{\omega} = \dot{H}$$

Wheel Dynamics

$$J(\dot{\Omega} + \ddot{\theta}) = -T_w = \dot{h}$$



Feedback Law, Choose

$$T_w = \underbrace{-K_p \theta}_{\text{Position feedback}} - \underbrace{K_r \dot{\theta}}_{\text{Rate feedback}}$$

Position feedback Rate feedback

Stabilize RIGID BODY

Then:

$$\ddot{\theta} + (K_r / I)\dot{\theta} + (K_p / I)\theta = 0 \quad \rightarrow \text{Laplace Transform}$$

$$s^2 + (K_r / I)s + (K_p / I) = 0$$

Characteristic Equation

$$s^2 + 2\zeta\omega s + \omega^2 = 0$$

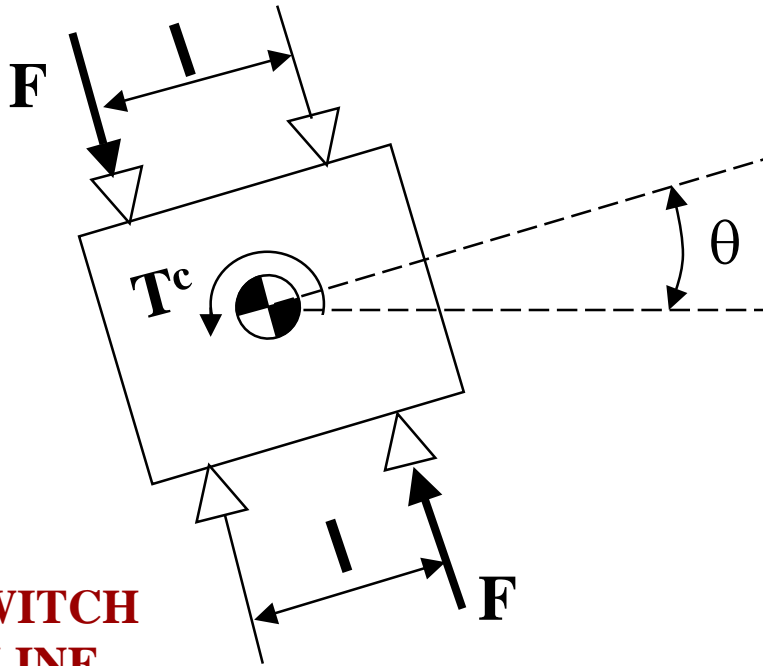
Nat. frequency

$$\omega = \sqrt{K_p / I}$$

$$\zeta = K_r / (2\omega I)$$



Jet Control Example (1)



Introduce control torque
force couple from jet thrust

$$I\ddot{\theta} = T^c$$

Only three possible values for T^c

$$T^c = \begin{cases} Fl & \text{if } \theta < 0 \\ 0 & \text{if } \theta = 0 \\ -Fl & \text{if } \theta > 0 \end{cases}$$

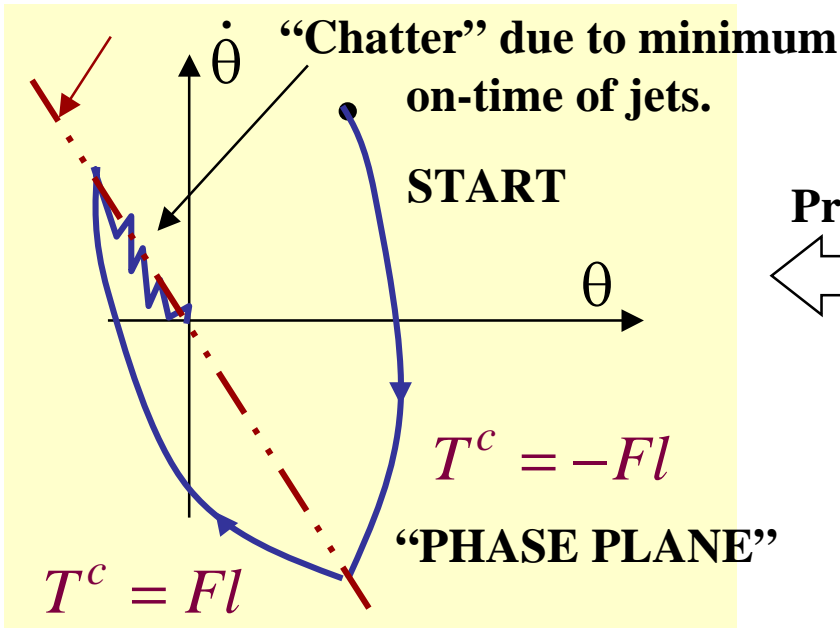
Can stabilize (drive θ to 0) by feedback law:

$$T^c = -Fl \cdot \text{sgn}(\theta + \tau \dot{\theta})$$

Where

$$\text{sgn}(x) = \frac{x}{|x|} \quad \tau = \text{time constant}$$

SWITCH LINE



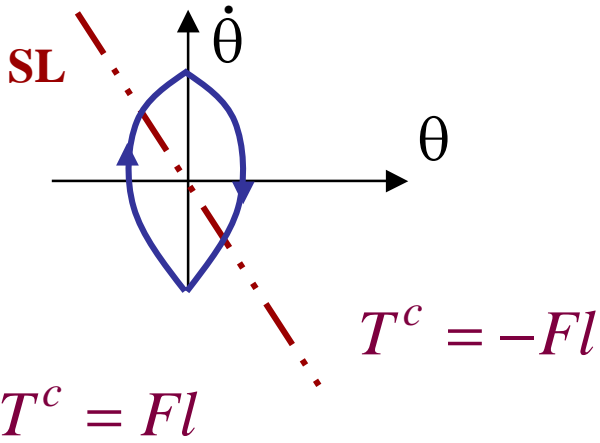
Problem

“PHASE PLANE”



Jet Control Example (2)

“Chatter” leads to a “limit cycle”, quickly wasting fuel

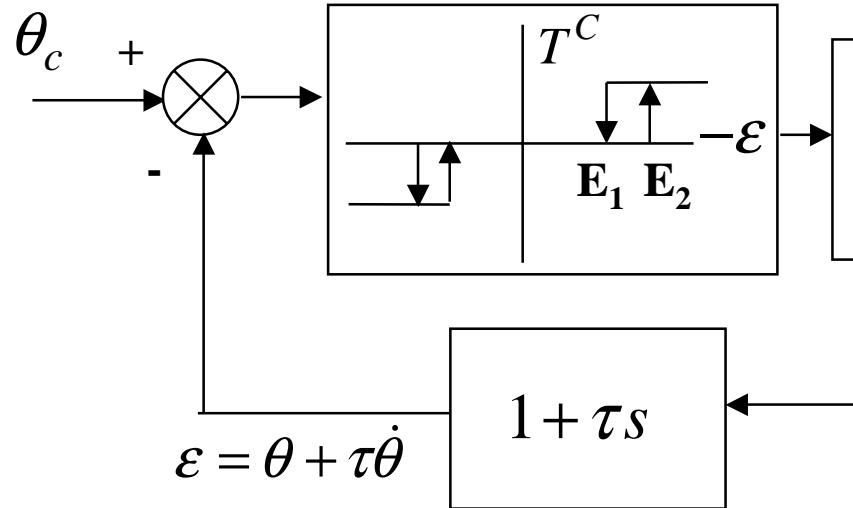


“PHASE PLANE”

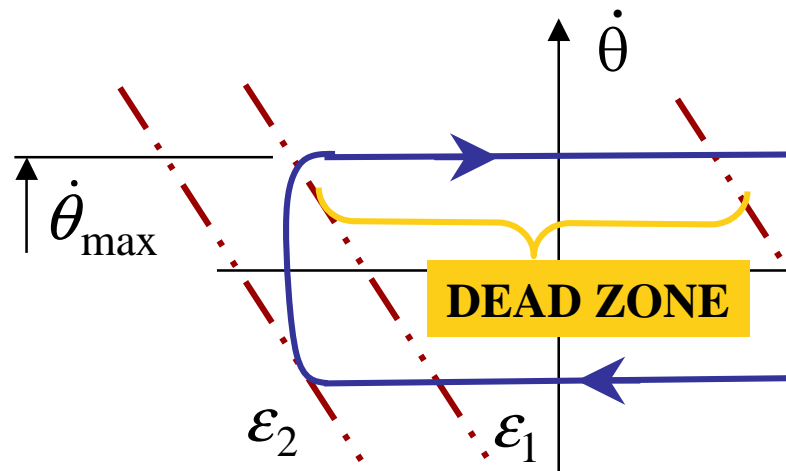
At Switch Line: $\theta + \tau\dot{\theta} = 0$

- Low Frequency Limit Cycle
- Mostly Coasting
- Low Fuel Usage
- θ and $\dot{\theta}$ bounded

Solution:
Eliminate “Chatter” by “Dead Zone” ; w



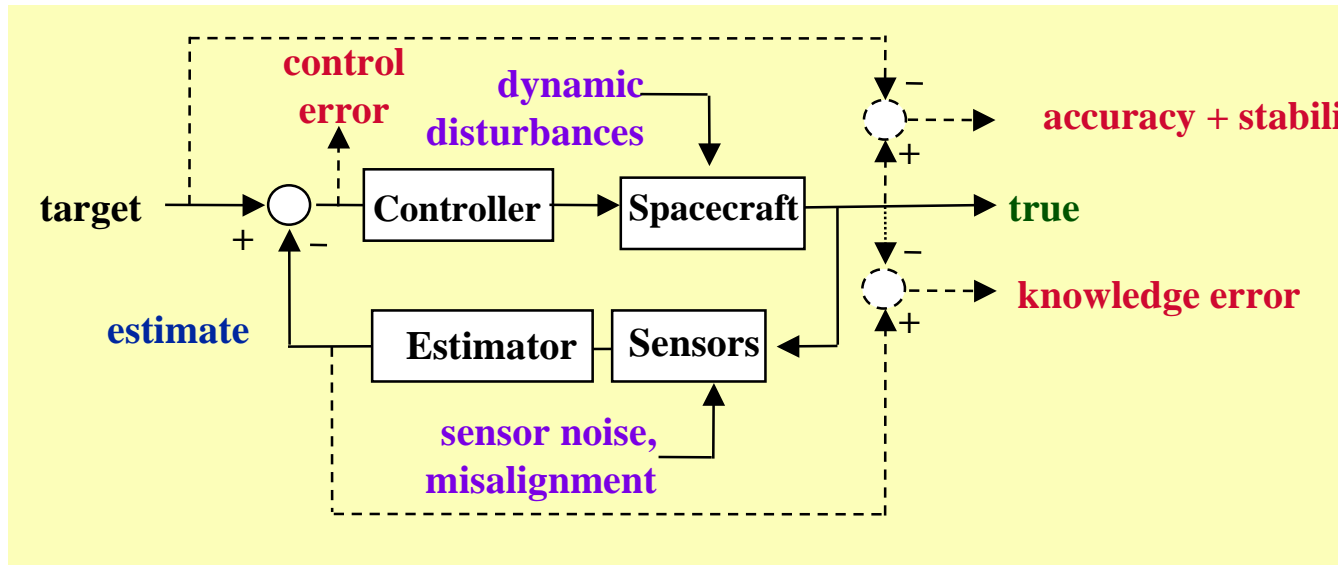
Results in the following motion





ACS Block Diagram (2)

In the “REAL WORLD” things are somewhat more complicated

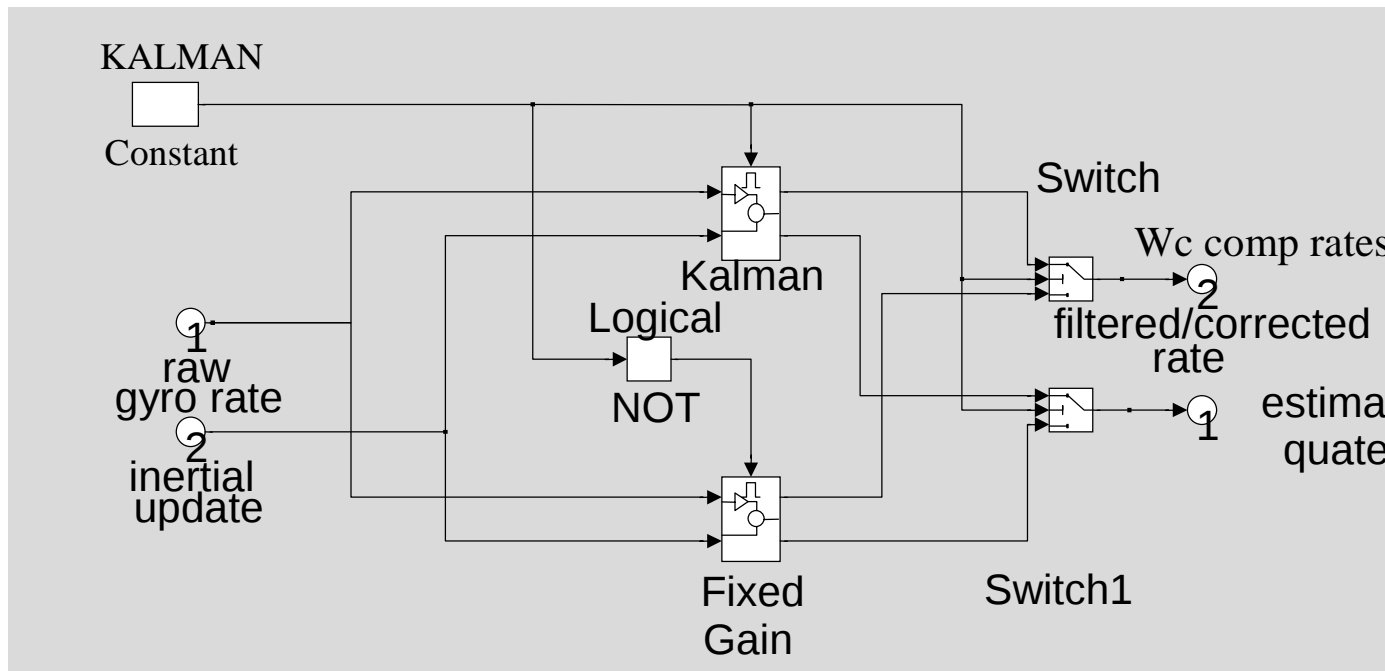


- Spacecraft not a RIGID body, sensor , actuator & avionic
- Digital implementation: work in the z-domain
- Time delay (lag) introduced by digital controller
- A/D and D/A conversions take time and introduce errors: 16-bit electronics, sensor noise present (e.g rate gyro @ D
- Filtering and estimation of attitude, never get q directly



Attitude Determination

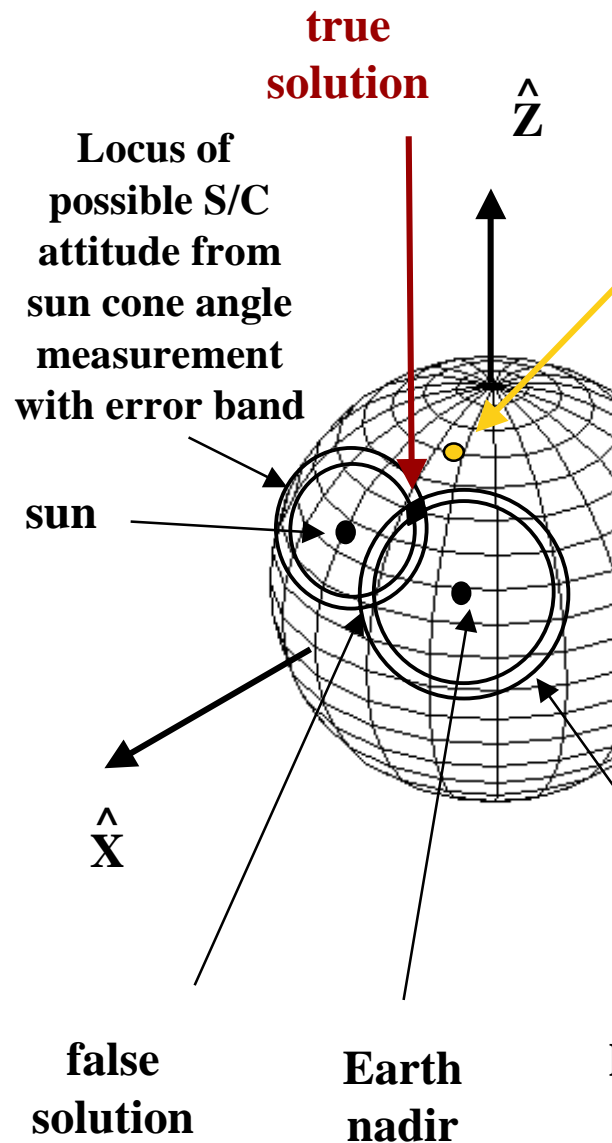
- Attitude Determination (AD) is the process of deriving of spacecraft attitude from (sensor) measurement data. Exact determination is NOT POSSIBLE, always have some error
- Single Axis AD: Determine orientation of a single spacecraft in space (usually spin axis)
- Three Axis AD: Complete Orientation; single axis (Euler when using Quaternions) plus rotation about that axis





Single-Axis Attitude Determination

- Utilizes sensors that yield an arc-length measurement between sensor boresight and known reference point (e.g. sun, nadir)
- Requires at least two independent measurements and a scheme to choose between the true and false solution
- Total lack of a priori estimate requires three measurements
- Cone angles only are measured, not full 3-component vectors. The reference (e.g. sun, earth) vectors are known in the reference frame, but only partially so in the body frame.





Three-Axis Attitude Determination

- Need two vectors (u, v) measured in the spacecraft frame and known in reference frame (e.g. star position on the celestial sphere)
- Generally there is redundant data available; can extend the calculations on this chart to include a least-squares estimate for the attitude
- Do generally not need to know absolute values

Define:

$$\hat{i} = u / |u|$$

$$j = (u \times v) / |u \times v|$$

$$\hat{k} = \hat{i} \times \hat{j}$$

Want Attitude Matrix

$$\underbrace{\begin{bmatrix} \hat{i}_B & \hat{j}_B & \hat{k}_B \end{bmatrix}}_M = T \cdot \underbrace{\begin{bmatrix} \hat{i}_R \end{bmatrix}}_N$$

$$|u|, |v|$$

So: $T = MN^{-1}$

Note: N must be non-singular (= full rank)



Effects of Flexibility (Spinners)

The previous solutions for Euler's equations were only valid for a **RIGID BODY**. When flexibility exists, energy dissipation will occur.

$$\underline{H} = \underline{I} \underline{\omega} \quad \longrightarrow \quad \text{CONSTANT}$$

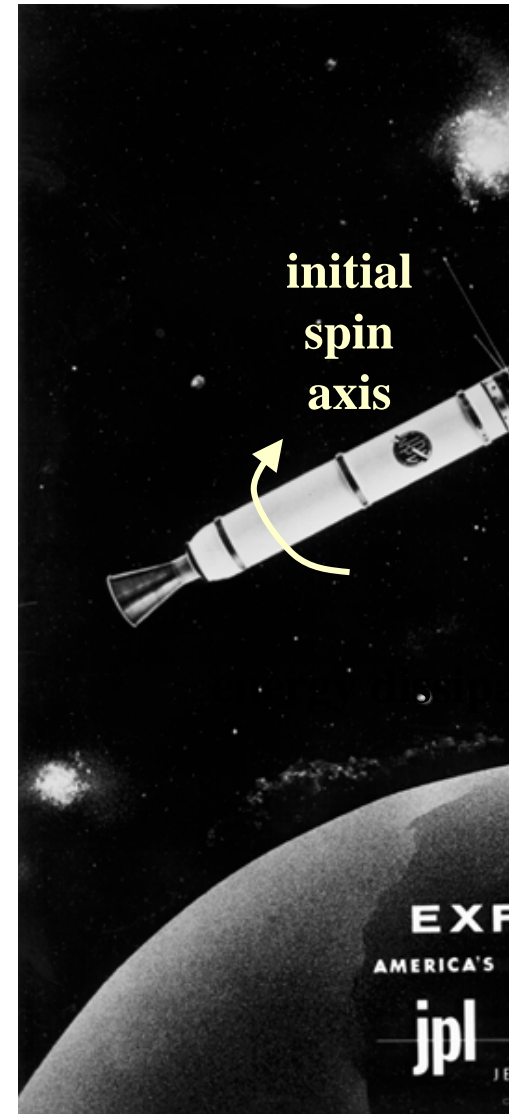
Conservation of
Angular Momentum

$$E_{\text{ROT}} = \frac{1}{2} \underline{\omega}^T \underline{I} \underline{\omega} \quad \longrightarrow \quad \text{DECREASING}$$

∴ Spin goes to maximum
I and minimum ω

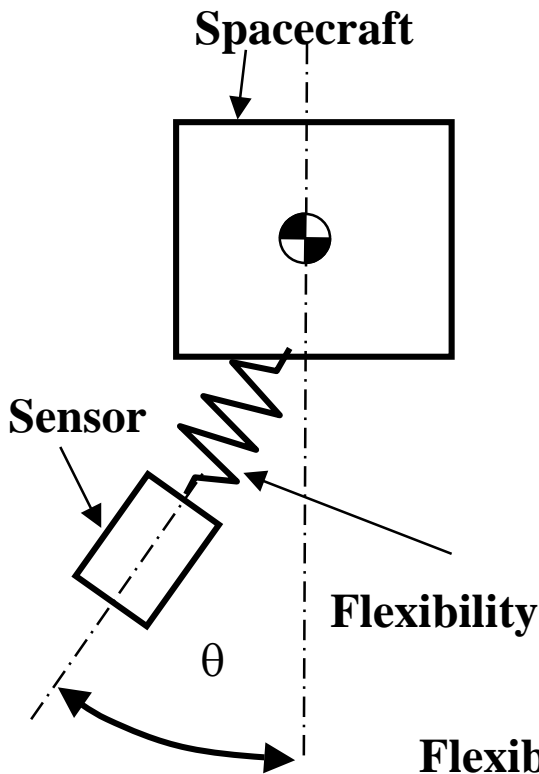
CONCLUSION: Stable Spin is only possible about the axis of maximum inertia.

Classical Example: **EXPLORER 1**



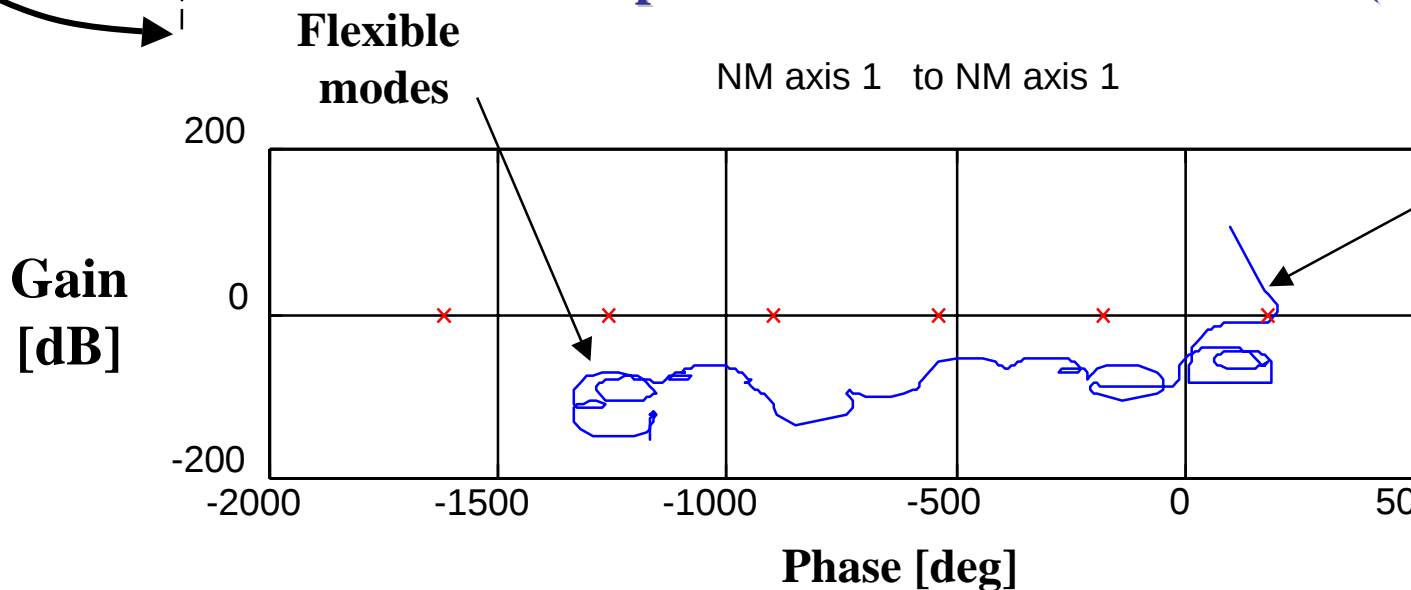


Controls/Structure Interaction



- Can't always neglect flexible modes (arrays, sunshield)
- Sensor on flexible structure, modes cause phase loss
- Feedback signal "corrupted" by flexible deflections; can become unstable
- Increasingly more important as spacecraft become larger and pointing goals become more stringent

Loop Gain Function: Nichols Plot (Nyquist)





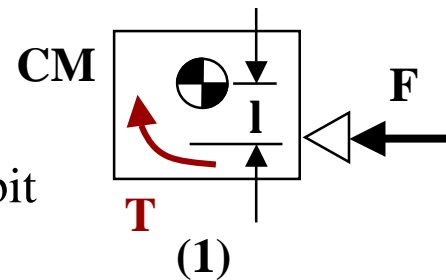
Other System Considerations (1)

- Need on-board COMPUTER
 - Increasing need for on-board performance and autonomy
 - Typical performance (somewhat outdated: early 1990's)
 - 35 pounds, 15 Watts, 200K words, 100 Kflops/sec, CMOS
 - Rapidly expanding technology in real-time space-based computing
 - Nowadays get smaller computers, rad-hard, more MIPS
 - Software development and testing, e.g. SIMULINK Real Time compilation from development environment MATLAB C, C++ processor is getting easier every year. Increased attention on software
- Ground Processing
 - Typical ground tasks: Data Formatting, control functions, data analysis
 - Don't neglect; can be a large program element (operations)
- Testing
 - Design must be such that it can be tested
 - Several levels of tests: (1) benchtop/component level, (2) environmental testing (vibration, thermal, vacuum), (3) ACS tests: air bearing, simulation with part hardware, part simulated



Other System Considerations (2)

- Maneuvers
 - Typically: Attitude and Position Hold, Tracking/Slewing, SAFE
 - Initial Acquisition maneuvers frequently required
 - Impacts control logic, operations, software
 - Sometimes constrains system design
 - Maneuver design must consider other systems, I.e.: solar arrays towards sun, radiators pointed toward space, antennas toward Earth
- Attitude/Translation Coupling
 - (1) Δv from thrusters can affect attitude
 - (2) Attitude thrusters can perturb the orbit
- Simulation
 - Numerical integration of dynamic equations of motion
 - Very useful for predicting and verifying attitude performance
 - Can also be used as “surrogate” data generator
 - “Hybrid” simulation: use some or all of actual hardware, digital the spacecraft dynamics (plant)
 - can be expensive, but save money later in the program





Future Trends in ACS Design

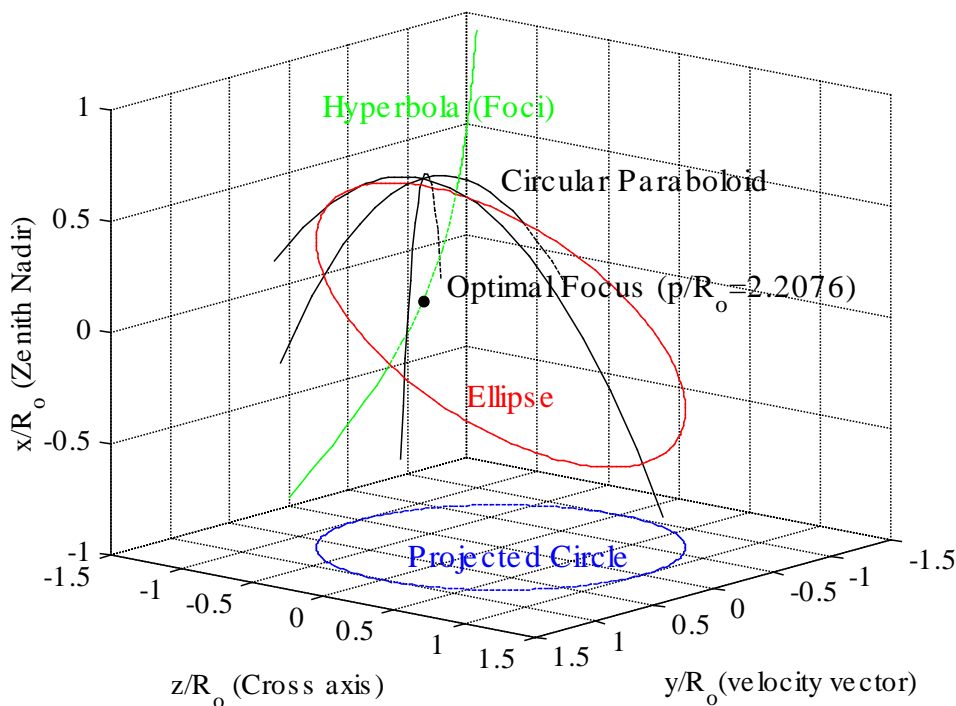
- Lower Cost
 - Standardized Spacecraft, Modularity
 - Smaller spacecraft, smaller Inertias
 - Technological progress: laser gyros, MEMS, magnetic wheel b
 - Greater on-board autonomy
 - Simpler spacecraft design
- Integration of GPS (LEO)
 - Allows spacecraft to perform on-board navigation; functions in
 - from ground station control
 - Potential use for attitude sensing (large spacecraft only)
- Very large, evolving systems
 - Space station ACS requirements change with each added modu
 - Large spacecraft up to 1km under study (e.g. TPF Able “kilotru
 - Attitude control increasingly dominated by controls/structure in
 - Spacecraft shape sensing/distributed sensors and actuators



Advanced ACS concepts

Visible Earth Imager using a Distributed Satellite System

- No ΔV required for collector spacecraft
- Only need ΔV to hold combiner spacecraft at paraboloid's focus



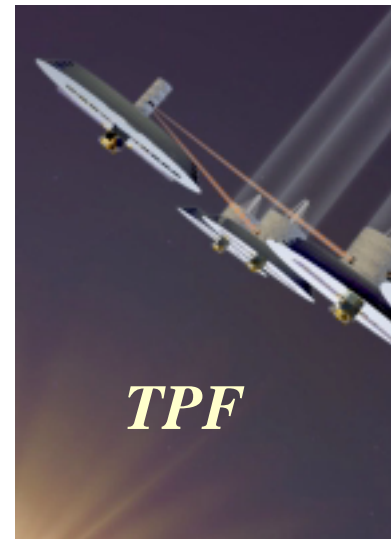
Formation Flying

- Exploit natural orbital dynamics to synthesize sparse arrays using formation flying
- Hill's equations exploit "orbit ellipse" solution

$$\ddot{x} - 2\dot{y}_n - \ddot{z}$$

$$\ddot{y} + 2\dot{x}_n$$

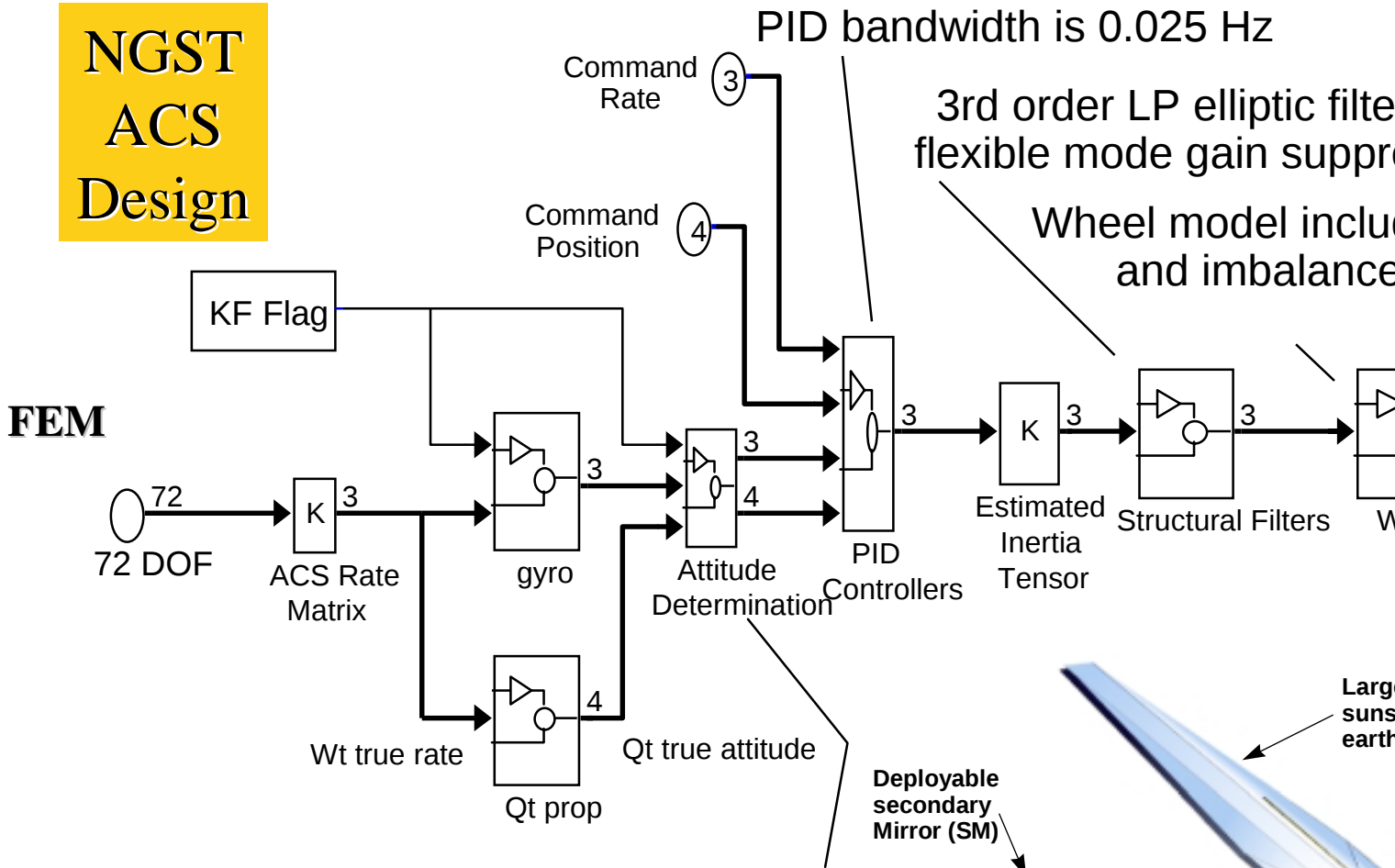
$$\ddot{z} + \dots$$



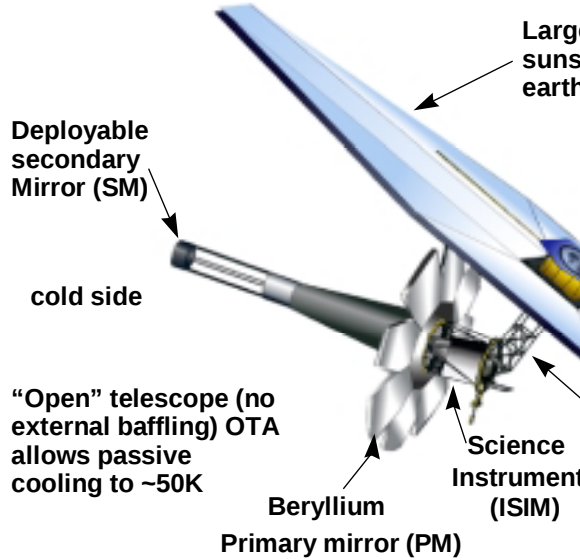


ACS Model of NGST (large, flexible S/

NGST ACS Design

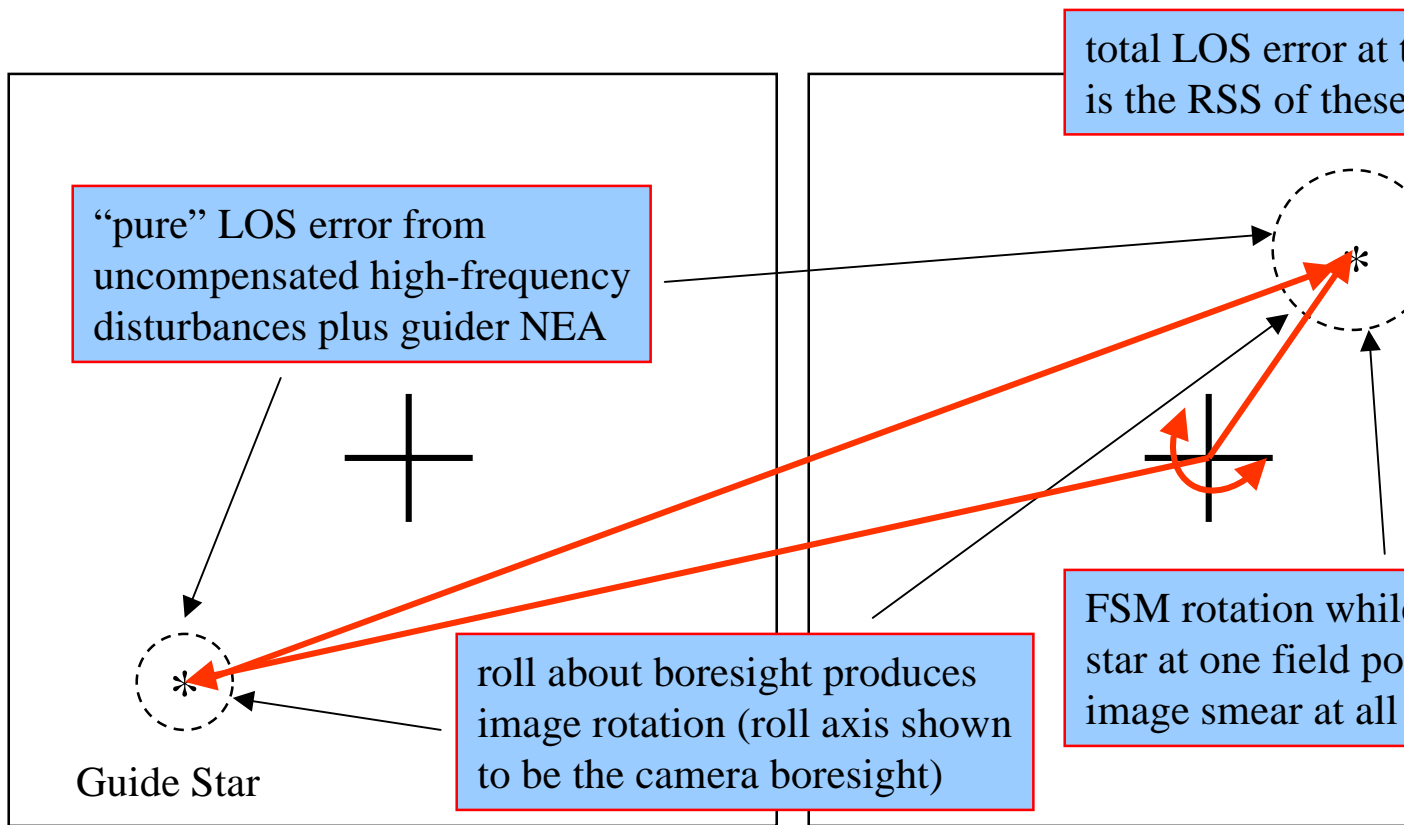


Kalman Filter blends 10 Hz IRU and 2 Hz ST data to provide optimal attitude estimate; option exists to disable the KF and inject white noise, with amplitude given by steady-state KF covariance into the controller position channel





Attitude Jitter and Image Stability



Source: G. Mosier
NASA GSFC

Guider

Camera

Important to assess impact of attitude jitter (“stability”) on image quality. Can compensate with fine pointing system. Use a guider camera as sensor and a 2-axis FSM as actuator.

R
I
RMS

E.g. HST: RMS LOS =



References

- James French: AIAA Short Course: “Spacecraft Systems Engineering”, Washington D.C., 1995
- Prof. Walter Hollister: 16.851 “Satellite Engineering” Course, Fall 1997
- James R. Wertz and Wiley J. Larson: “Space Mission Analysis and Design”, Second Edition, Space Technology Series, Space Systems Library, Microcosm Inc, Kluwer Academic Publishers